MASON

Electrical Constants of Steel Conductors for Transmission Lines

Electrical Engineering

M. S.

1913

UNIV. OF
ILLINOIS
LIBRARY

THE UNIVERSITY
OF ILLINOIS
LIBRARY
1913
M38

ELECTRICAL CONSTANTS OF STEEL CONDUCTORS FOR TRANSMISSION LINES

BY

MAYNE SEGUINE MASON
B. S. University of Illinois, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1913

1913
M25

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31 1913

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Mayne Seguine Mason

ENTITLED

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Master of Science in Electrical Engineering

Ernst Berg
In Charge of Major Work

Ernst Berg
Head of Department

Recommendation concurred in:

Morgan Brooks
J. M. Bryant
Ellery B. Paine

Committee on Final Examination

246497

UIUC

TABLE OF CONTENTS

I.	INTRODUCTION	1 - 2
II.	MATHEMATICAL THEORY	3
III.	EXPERIMENTAL APPARATUS	4 - 8
IV.	EXPERIMENTAL RESULTS	9 - 67
V.	CONCLUSIONS	68 - 69
	APPENDIX I	70 - 90

Digitized by the Internet Archive
in 2014

http://archive.org/details/electricalconsta00maso

ELECTRICAL CONSTANTS OF STEEL CONDUCTORS FOR TRANSMISSION LINES.

I INTRODUCTION

Copper has been practically the only material used in conductors for the transmission of electric energy because of its high electric conductivity. With the introduction of high voltage power transmission the demand for high conductivity of the conductor has become less insistent, and the attention of designers has been called to the mechanical strength of line necessary for long spans. As a result, steel conductors have been used in several installations.

It can be shown experimentally and mathematically that when alternating current is passed through a conductor it does not distribute itself uniformly over the cross-section, but that the current density is greater in the outer layers of the wire than in the central portion. Since the inner portion does not carry its share of current the effective cross-section is reduced, and the resistance is increased. Even at the comparatively low frequencies used in commercial work this increase of resistance, or skin effect, is important in conductors of steel or other magnetic materials.

The solution of the equations governing the establishment and distribution of electric currents in a cylindrical conductor has been considered by Maxwell, Oliver Heaviside, Lord Kelvin, Lord Rayleigh, Dr. A. Russell, and others. The mathematical

theory of this thesis presents the deduction of Maxwell's equations for skin effect, with the modifications effected by Sir William Thomson with his ber and bei expressions.

While the primary object of this investigation has been to obtain commercial constants, the results are of scientific value in that they present for the first time, as far as the author's knowledge goes, a fairly complete statement of the magnetic state of the conductor, giving as they do the relation between the permeability, current density, frequency, and size of conductor.

II MATHEMATICAL THEORY

This thesis is founded on the expression for skin effect given by Lord Kelvin:

$$K = \frac{q}{2} \frac{\text{ber } q \text{ bei' } q - \text{ber' } q \text{ bei } q}{(\text{ber' } q)^2 + (\text{bei' } q)^2} .$$

Since there is considerable difficulty even for physicists and engineers in following all the steps of the theory as presented by Maxwell and extended by Lord Kelvin, a complete development is presented in Appendix I.

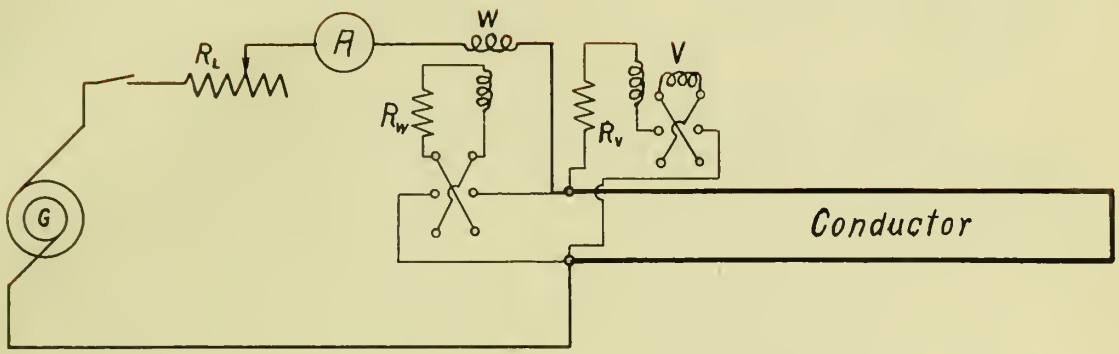


Diagram of Connections

III.

EXPERIMENTAL APPARATUS.

The conductors tested in this experiment were:

3/16 inch solid steel, galvanized

7/32 inch stranded steel, galvanized

3/8 inch stranded steel, galvanized

No. 2 B & S copper-clad steel.

Dimensions and ohmic resistance of each conductor are given on pages (61-63).

Each conductor was bent so as to form a parallel return loop, the distance from inner surface of the conductor to the inner surface of the return conductor being six inches.

Variable room temperature and large line currents caused variable ohmic resistance. The first plan was to observe the conductor temperature by means of several thermometers spaced at equal intervals with the mercury bulbs placed next to the conductor and covered with a thin layer of putty. This method failed in two respects: the temperature changed while the several thermometers were being read, and the putty covering, by its absorption and retention of heat, caused a lag in the indications of the thermometers. The fall of potential method of measuring resistance was finally adopted. With a direct current ammeter and low reading voltmeter the data for calculation of ohmic resistance was obtained. In case the resistance has changed, time was given for the conductor to cool or else it was heated with a large alternating current until the average resistance was obtained again.

The power at 25 and 40 cycles was furnished by a Thomson Houston smooth core generator, No. 683, Class A 35, 1100 volts, 1500 R.P.M. The voltage was stepped down to about 60 volts, through a General Electric transformer, No. 745828, Type H60, Form K, 1100/2200 110/220 volts, 5 K. W., with the high tension coils in parallel and the low tension coils in series. Rheostats in series with the conductor gave line current control, and rheostatic control of the armature and field current of the driving motor assured constant frequency. Since the motor could not attain the speed necessary for 90 cycles, both the 60 and 90 cycle runs were made on General Electric generator No. 95584, Type AHB, Class 6-7.5, Form A, 20 amperes, 220 volts, 1200 R.P.M. The six armature coils are spaced 30 electrical degrees apart on the armature. No transformer was used in this set up, rheostats in the generator field circuit giving the necessary voltage regulation. The smooth core generator gave a sine voltage wave; the wave from the General Electric generator was also very free from harmonics because of the series connection of the armature coils.

For high frequency (500-2000 cycles) General Electric Rotary Converter No. 352411, Type Sc-80-1.5-3000, 2000 cycles, 3000 R.P.M., A.C.Voltage 43, Amperes 35; D. C. Voltage 150, Amperes 17, was used. A two-pole a. c. magneto, direct-connected to the shaft of the rotary converter gave frequency indications which were 1/40 the frequency of the rotary converter. This arrangement made the same frequency meter available for low and high frequency tests inasmuch as the 500 to 2000 cycles were indicated as 5--/40 to 2000/40, or 12.5 to 50 cycles. Rheostats

in the armature and the field circuit gave control of the frequency. The machine is described and illustrated in the General Electric Review, Vol. XVI, No. 1, January 1913. Its construction assures a sine voltage wave, and oscillograms verify this claim.

The frequency meter used was a Biddle instrument of the vibrating reed type.

The power consumed by the conductor was measured by General Electric wattmeter dynamometer No. 181928. The sensibility of the instrument was a deflection of 500 millimeters on a scale one meter from the mirror for a power measurement of .128 watts. For measurements of power greater than this resistances were put in series with the potential coil of the instrument. These resistances ranged in value from 0 to 60,000 ohms. Since the instrument was not direct reading it was necessary to calibrate it for each value of resistance used. All the calibrations were made with direct current power and checked from time to time during the test so as to detect any possible disturbance of the instrument. The zero reading of the dynamometer was obtained frequently as a precaution against accidental movement of the instrument with a consequent change in calibration due to a different relative position of the suspended and fixed coils. To make the deflections independent of the zero indication, for each value of line current a reading was taken with the deflection to the right, then the current in the potential coil was reversed and the deflection

to the left was observed. One-half the total deflection from left to right was used in the determinatcn of watts indicated. No single deflection from zero to right or to left exceeded 250 millimeters nor was less than 30 millimeters, since this range gave indications of greatest accuracy.

The potential drop across the conductor was measured by General Electric voltmeter dynamometer No. 181929. The sensibility of the instrument without external resistance was approximately 1.08 volts for a 500 millimeter deflection on a scale one meter distant. For higher voltages resistances ranging from 0 to 60,000 ohms were connected in series with the voltmeter. The same precautions were taken as with the dynamometer wattmeter and the same method of calibration used. The reactance of the voltmeter was determined by a series of high frequency tests, checked by calculation of reactance from design data, and the correcti n for reactance drop in the voltmeter applied to each voltage reading. Only in high frequency tests with very low resistance in series with the voltmeter was this correction appreciable. Readings of deflections both to right and to left of zero were made as in the case of the wattmeter.

The current was measured with Weston portable ammeters designed for frequencies under 133 cycles per second. Each ammeter was calibrated with G. E. dynamometer ammeter #181928 at 500, 1000, 1500, and 2000 cycles and found accurate within one per cent. At frequent intervals during the tests the calibration of each ammeter was checked with a standard instrument.

IV

EXPERIMENTAL RESULTS

The results of the experimental tests are given in the form of tables and curves, extending from page (9) to page (60). In the hyphenated titles of the curves the first word or symbol refers to the ordinates of the curve, the second to the abscissas.

From the observed values of watts consumed and line current the effective resistance R ($= \frac{W}{I^2}$), was calculated for each value of line current and the values plotted. From a smooth curve drawn through the points, corrected values of effective resistance R'_E were obtained, and by the aid of the K-q curves on page (58) permeability values were calculated. Permeability-current curves serve as a check on R readings.

From the observed voltage E the effective impedance Z_E $\left(= \frac{E}{I}\right)$ was calculated, and a $Z_E - I$ curve plotted. From this curve corrected values of Z'_E were taken. The inductance was calculated thus: $Z'^2_E - R'^2_E = X^2$. Reactance - line current curves were plotted for each frequency. From these curves readings were taken for the reactance at different frequencies and the same line current, giving an X-frequency curve at a certain current density. This process was repeated for various current densities giving a series of X-frequency curves.

For each frequency a $K - I_{\square}''$ curve was plotted. (K=skin effect coefficient, $I_{\square}''$ = current density in amperes per square inch. From these curves values were taken for

K at different frequencies and the same current density. This was repeated for different current densities giving a series of K - frequency curves.

Sample calculations are given from page (61) to page (66).

TABLE I.

3/16 INCH SOLID STEEL CONDUCTOR. 25 CYCLES.

I	I_0''	W	R_E	R_E'	K	q	μ	E	Z_E	Z_E'	X	I
1	35.3			.1410	1.032	1.57	271					1
1.5	52.9			.1442	1.056	1.81	360					1.5
2	70.6			.1482	1.085	2.03	454	.249	.125	.160	.0603	2
3	105.8	1.46	.1622	.158	1.157	2.42	645	.513	.171	.170	.0627	3
4	141.2	2.74	.1712	.1716	1.256	2.81	869			.1836	.0653	4
5	176.3	4.66	.1864	.1875	1.375	3.15	1095	.99	.198	.199	.0667	5
6	211.6	7.40	.2055	.2040	1.495	3.48	1335			.216	.071	6
7	247	10.24	.2126	.2130	1.560	3.66	1475	1.60	.2286	.2265	.077	7
8	282	13.80	.2156	.2175	1.595	3.76	1560			.234	.0863	8
9	317	17.60	.2170	.2205	1.615	3.82	1610	2.15	.239	.2388	.0917	9
10	353	22.30	.2230	.2225	1.640	3.86	1640	2.42	.242	.2419	.0949	10
11	388			.2225	1.640	3.86	1640			.2422	.0957	11
12	423	32.10	.2230	.2220	1.625	3.84	1625	2.91	.2426	.2422	.0968	12
13	459			.2190	1.605	3.78	1575			.2402	.0987	13
14	494	42.40	.2160	.2160	1.580	3.71	1515	3.30	.2357	.237	.0976	14
15	529	47.40	.2110	.2113	1.550	3.63	1450	3.47	.2313	.232	.0958	15

TABLE II.

3/16 INCH SOLID STEEL CONDUCTOR. 40 CYCLES.

I	$I_{0''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
1	35.3			.1460	1.068	1.90	249					1
1.5	52.9			.1500	1.098	2.11	306	.245	.1633			1.5
2	70.6	.62	.1550	.1550	1.135	2.31	368			.1693	.0681	2
3	105.8	1.48	.1645	.1671	1.223	2.68	495			.1827	.0739	3
4	141.2	2.92	.1825	.1840	1.347	3.08	654	.826	.2065	.201	.081	4
5	176.3	5.24	.2096	.2090	1.530	3.58	880	1.16	.232	.2273	.0892	5
6	211.6	8.52	.2366	.2330	1.705	4.06	1135	1.56	.260	.2535	.0999	6
7	247	11.90	.2430	.2470	1.810	4.36	1310	1.90	.2715	.2708	.1110	7
8	282	16.30	.2548	.2559	1.865	4.51	1400	2.23	.279	.280	.1156	8
9	317			.2590	1.895	4.59	1450			.2848	.1185	9
10	353	25.80	.2580	.2609	1.910	4.63	1475	2.87	.287	.2867	.1191	10
11	388			.2610	1.910	4.63	1475			.2862	.1175	11
12	423			.2596	1.900	4.61	1465			.2856	.119	12
13	459	43.00	.2543	.2573	1.885	4.57	1435	3.67	.2822	.2835	.119	13
14	494			.2540	1.860	4.50	1395			.2803	.1185	14
15	529	56.30	.2494	.2494	1.825	4.40	1330	4.15	.2767	.2768	.1201	15

TABLE III.

3/16 INCH SOLID STEEL CONDUCTOR. 60 CYCLES.

I	$I_{\square''}$	W	R_E	R'_E	K	q	μ	E	z_E	z'_E	X	I
1	35.3	.38	.1688	.1552	1.136	2.32	247					1
1.5	52.9			.1613	1.131	2.52	291	.254	.1693			1.5
2	70.6	.73	.1824	.1688	1.236	2.74	344	.3556	.178	.1843	.074	2
3	105.8	.63	.1834	.1872	1.370	3.14	452			.2048	.083	3
4	141.2	3.48	.2174	.2120	1.552	3.64	607	.910	.2275	.231	.0917	4
5	176.3	6.50	.2600	.2430	1.780	4.26	830	1.31	.262	.2659	.108	5
6	211.6	9.80	.2720	.2715	1.990	4.85	1080	1.85	.3085	.2985	.124	6
7	247	14.30	.2920	.2885	2.110	5.22	1250	2.25	.3215	.3205	.139	7
8	282	19.10	.2985	.2983	2.185	5.42	1345	2.67	.3338	.3337	.1495	8
9	317			.3035	2.220	5.53	1400			.3392	.1511	9
10	353	30.70	.3070	.3060	2.240	5.60	1435	3.41	.341	.341	.1505	10
11	388			.3063	2.245	5.60	1435			.3414	.1508	11
12	423			.3055	2.235	5.55	1410			.3402	.1496	12
13	459	51.50	.3046	.3033	2.220	5.50	1385	4.40	.3384	.338	.1488	13
14	494			.3000	2.195	5.42	1345			.3353	.1498	14
15	529	67.00	.2980	.2955	2.165	5.35	1310	4.96	.3306	.332	.1513	15

TABLE IV.

3/16 INCH SOLID STEEL CONDUCTOR. 75 CYCLES.

I	$I_{0''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
1	35.3			.1637	1.195	2.58	245					1
1.5	52.9			.1715	1.252	2.80	288	.285	.190			1.5
2	70.6	.73	.1825	.1803	1.316	3.00	331	.405	.2025	.199	.0842	2
3	105.8	1.83	.2033	.2028	1.480	3.45	438	.676	.2253	.225	.0969	3
4	141.2	3.71	.2320	.2298	1.677	3.985	584	.988	.2470	.258	.1173	4
5	176.3	6.48	.2590	.2628	1.918	4.655	796	1.483	.2966	.296	.1361	5
6	211.6	10.70	.2974	.2960	2.160	5.33	1043	2.005	.3342	.334	.1547	6
7	247	15.60	.3185	.3143	2.294	5.71	1197	2.48	.3544	.355	.165	7
8	282	20.70	.3235	.3250	2.372	5.94	1297	2.95	.3688	.3688	.1743	8
9	317			.3298	2.407	6.03	1336			.374	.1764	9
10	353	33.30	.3330	.3318	2.421	6.08	1359	3.75	.3750	.376	.1768	10
11	388			.3323	2.426	6.09	1362			.3765	.177	11
12	423			.3315	2.420	6.075	1357			.3753	.1759	12
13	459	55.50	.3284	.3290	2.400	6.01	1327	4.83	.3716	.3726	.1746	13
14	494			.3260	2.380	5.96	1306			.370	.175	14
15	529	72.30	.3210	.3220	2.350	5.87	1266	5.54	.369	.3665	.175	15

TABLE V.

3/16 INCH SOLID STEEL CONDUCTOR. 500 CYCLES.

I	$I_{\square''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
1	35.3	.324	.324	.322	2.282	5.69	184	.423	.423	.409	.252	1
2	70.6	1.43	.358	.366	2.595	6.56	244	.904	.452	.467	.290	2
3	105.8	3.87	.430	.428	3.036	7.81	346			.544	.336	3
4	141.2	7.75	.484	.500	3.547	9.26	486	2.55	.638	.638	.3915	4
5	176.3	14.8	.592	.590	4.184	11.07	696	3.75	.750	.750	.463	5
6	211.6	24.3	.675	.666	4.724	12.6	901	5.06	.843	.844	.518	6
7	247	35.2	.718	.718	5.092	13.62	1052	6.32	.903	.901	.544	7
8	282	47.3	.739	.748	5.305	14.22	1148	7.38	.923	.929	.551	8
9	317	62.4	.771	.769	5.46	14.67	1220	8.52	.947	.947	.553	9
10	353	77.2	.772	.780	5.53	14.86	1254	9.53	.953	.953	.548	10
11	388			.783	5.555	14.95	1269			.956	.548	11
12	423	112.7	.783	.787	5.58	15.01	1280	11.44	.953	.953	.538	12

TABLE VI.

3/16 INCH SOLID STEEL CONDUCTOR. 1000 CYCLES.

I	$I_{a''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
.5	17.7	.103	.412									
1.0	35.3	.435	.435	.439	3.112	8.04	183	.638	.638	.546	.4726	1
2	70.6	2.06	.515	.499	3.54	9.24	242	1.454	.727	.702	.494	2
3	105.8	5.1	.567	.567	4.02	10.6	319	2.41	.803	.786	.544	3
4	141.2	10.1	.632	.647	4.59	12.21	423	3.51	.878	.889	.610	4
5	176.3	18.8	.752	.748	5.307	14.23	574	5.075	.015	1.015	.686	5
6	211.6			.857	6.08	16.42	765			1.133	.741	6
7	247			.938	6.65	18.01	920	8.63	1.232	1.220	.780	7
8	282	64.4	1.007	.987	7.00	19.02	1025	10.16	1.270	1.270	.799	8
9	317	81.4	1.005	1.02	7.24	19.70	1100	11.66	1.295	1.295	.798	9
10	353	103	1.03	1.04	7.38	20.11	1147	13.13	1.313	1.313	.802	10
11	388			1.053	7.47	20.36	1176			1.324	.803	11
12	423	152.6	1.059	1.06	7.52	20.49	1190	15.94	1.328	1.328	.800	12
13	459			1.06	7.52	20.49	1190			1.322	.790	13
14	494	206	1.051	1.049	7.44	20.36	1176	18.4	1.314	1.311	.786	14
15	529	231.5	1.029	1.04	7.38	20.12	1148	19.33	1.288	1.290	.763	15

TABLE VII.

3/16 INCH SOLID STEEL CONDUCTOR. 1500 CYCLES.

I	I_0''	W	R_E	R_E'	K	q	μ	M	Z_E	Z_E'	X	I
1	35.3	.488	.488	.529	3.752	9.85	183	.811	.811	.815	.620	1
1.5	52.9	1.30	.578									
2	70.6	2.38	.595	.595	4.22	11.17	236	1.821	.911	.896	.670	2
3	105.8	6.07	.674	.670	4.75	12.66	303	2.998	.999	.986	.723	3
4	141.2	12.1	.756	.763	5.41	14.52	399	4.42	1.105	1.101	.794	4
5	176.3	22.1	.884	.876	6.213	16.8	534	6.12	1.224	1.229	.862	5
6	211.6	38	1.055	1.014	7.19	19.56	723	8.42	1.403	1.370	.922	6
7	247	54.8	1.118	1.106	7.846	21.42	868	10.32	1.475	1.467	.964	7
8	282	75	1.171	1.169	8.29	22.68	972	12.23	1.529	1.528	.984	8
9	317	98	1.209	1.210	8.59	23.52	1047	14.16	1.573	1.567	.996	9
10	353	124	1.24	1.240	8.80	24.1	1098	15.92	1.592	1.593	1.000	10
11	388			1.259	8.93	24.5	1135			1.612	1.009	11
12	423	181.6	1.261	1.265	8.97	24.63	1147	19.49	1.624	1.623	1.011	12
13	459			1.262	8.95	24.6	1145			1.623	1.016	13
14	494	244	1.244	1.253	8.83	24.2	1108	22.68	1.620	1.619	1.025	14
15	529	279.9	1.243	1.242	8.81	24.14	1102	23.86	1.590	1.602	1.012	15

TABLE VIII.

3/16 INCH SOLID STEEL CONDUCTOR. 2000 CYCLES.

I	$I_{0''}$	W	R_E	R_E'	K	q	μ	E	Z_E	Z_E'	X	I
1	35.3	.549	.549	.600	4.256	11.26	180			.934	.716	1
1.5	52.9	1.46	.649					1.534	1.023			1.5
2	70.6	2.77	.692	.671	4.76	12.69	228	2.06	1.03	1.022	.771	2
3	105.8	6.56	.729	.751	5.32	14.27	289	3.377	1.126	1.128	.842	3
4	141.2	13	.813	.858	6.08	16.42	382	5.04	1.260	1.270	.936	4
5	176.3	24.8	.992	.987	7.00	19.02	513	7.11	1.421	1.440	1.048	5
6	211.6	42.3	1.175	1.121	7.95	21.7	668	9.49	1.581	1.580	1.113	6
7	247	60.7	1.238	1.225	8.69	23.8	804	11.74	1.678	1.678	1.146	7
8	282	82.6	1.290	1.291	9.16	25.13	896	14.09	1.761	1.750	1.181	8
9	317	107.6	1.328	1.336	9.47	26.03	960	16.12	1.791	1.790	1.191	9
10	353	136.5	1.365	1.370	9.72	26.72	1012	18.08	1.808	1.822	1.201	10
11	388			1.395	9.90	27.24	1053			1.842	1.203	11
12	423	204.2	1.417	1.410	10.00	27.53	1075	22.50	1.875	1.854	1.204	12
13	459			1.413	10.02	27.62	1082			1.860	1.209	13
14	494	276	1.408	1.411	10.01	27.58	1078	25.92	1.851	1.860	1.212	14
15	529	317	1.408	1.404	9.96	27.43	1068	28.04	1.869	1.858	1.217	15

TABLE VIII B.

K - I DATA FOR 3/16" SOLID STEEL CONDUCTOR.

$I_{\square''}$	I	25~ K	40~ K	60~ K	75~ K	500~ K	1000~ K	1500~ K	2000~ K
35.3	1	1.032	1.068	1.136	1.195	2.282	3.112	3.752	4.356
70.6	2	1.085	1.135	1.236	1.316	2.595	3.54	4.220	4.76
105.8	3	1.157	1.223	1.370	1.480	3.036	4.02	4.74	5.32
141.2	4	1.256	1.347	1.552	1.677	3.547	4.59	5.41	6.08
176.3	5	1.375	1.530	1.780	1.918	4.184	5.307	6.213	7.00
211.6	6	1.495	1.705	1.990	2.160	4.724	6.08	7.19	7.95
247	7	1.560	1.810	2.110	2.294	5.092	6.65	7.846	8.69
282	8	1.595	1.865	2.185	2.372	5.305	7.00	8.29	9.16
317	9	1.615	1.895	2.220	2.407	5.46	7.24	8.59	9.47
353	10	1.640	1.910	2.240	2.421	5.53	7.38	8.80	9.72
388	11	1.640	1.910	2.245	2.426	5.555	7.47	8.93	9.90
423	12	1.625	1.900	2.235	2.420	5.58	7.52	8.97	10.00
459	13	1.605	1.885	2.220	2.400		7.52	8.95	10.02
494	14	1.580	1.860	2.195	2.380		7.44	8.83	10.01
529	15	1.550	1.825	2.165	2.350		7.38	8.81	9.96

TABLE IX.

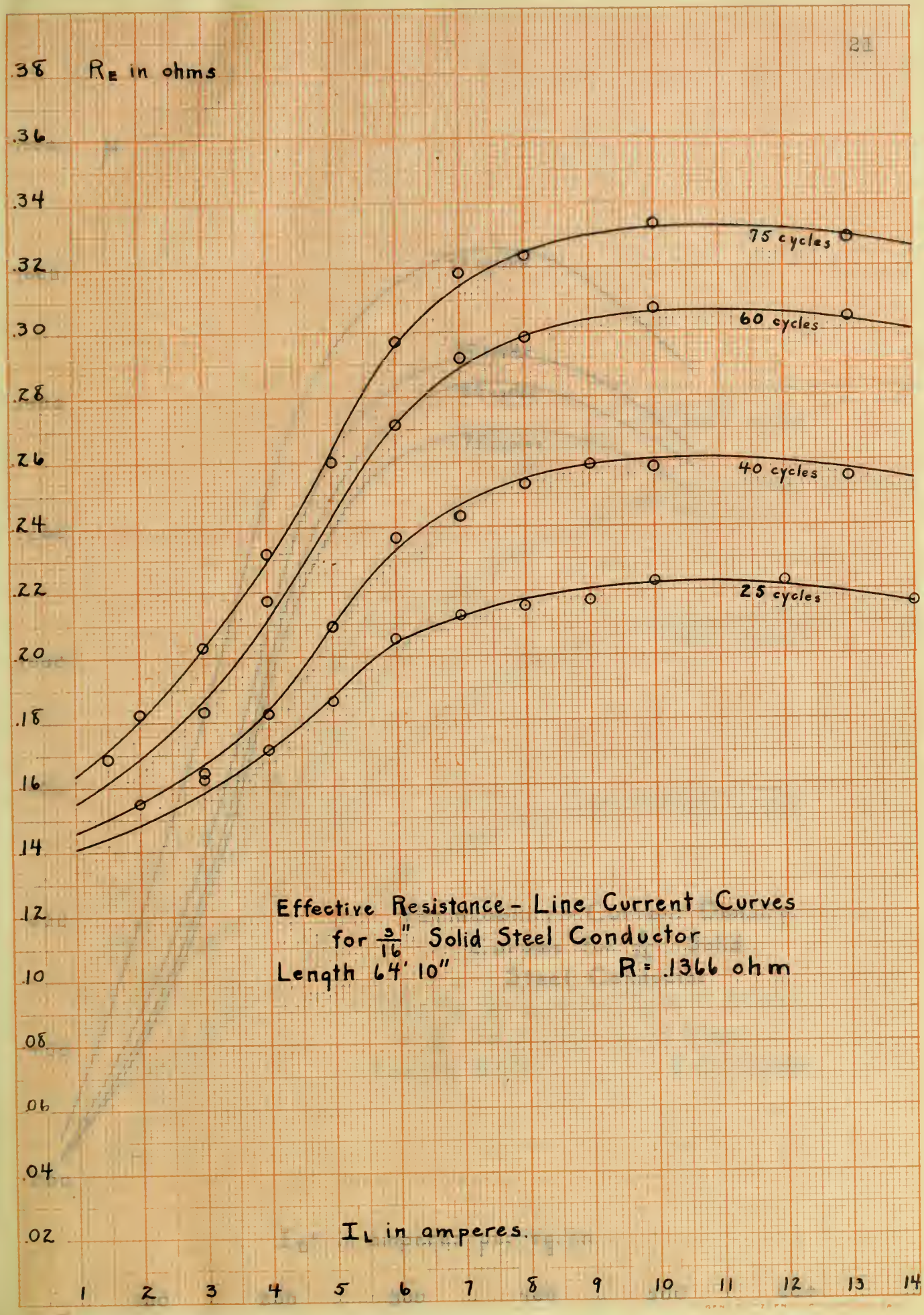
K - FREQUENCY DATA FOR 3/16" SOLID STEEL.

I_D"	I_L	25~ K	40~ K	60~ K	75~ K	500~ K	1000~ K	1500~ K	2000~ K
50	1.42	1.05	1.095	1.165	1.245	2.40	3.285	3.94	4.47
100	2.84	1.15	1.22	1.345	1.455	2.96	3.95	4.66	5.23
150	4.26	1.29	1.395	1.61	1.74	3.71	4.78	5.62	6.32
200	5.68	1.46	1.655	1.93	2.095	4.56	5.86	6.82	7.68
250	7.09	1.56	1.815	2.11	2.3	5.11	6.68	7.89	8.75
300	8.51	1.60	1.88+	2.20	2.40	5.40	7.13	8.46	9.33
400	11.34	1.64	1.905	2.245	2.43	5.625	7.50	8.95	9.94
495	14	1.58	1.85	2.19	2.38		7.44	8.83	10.01

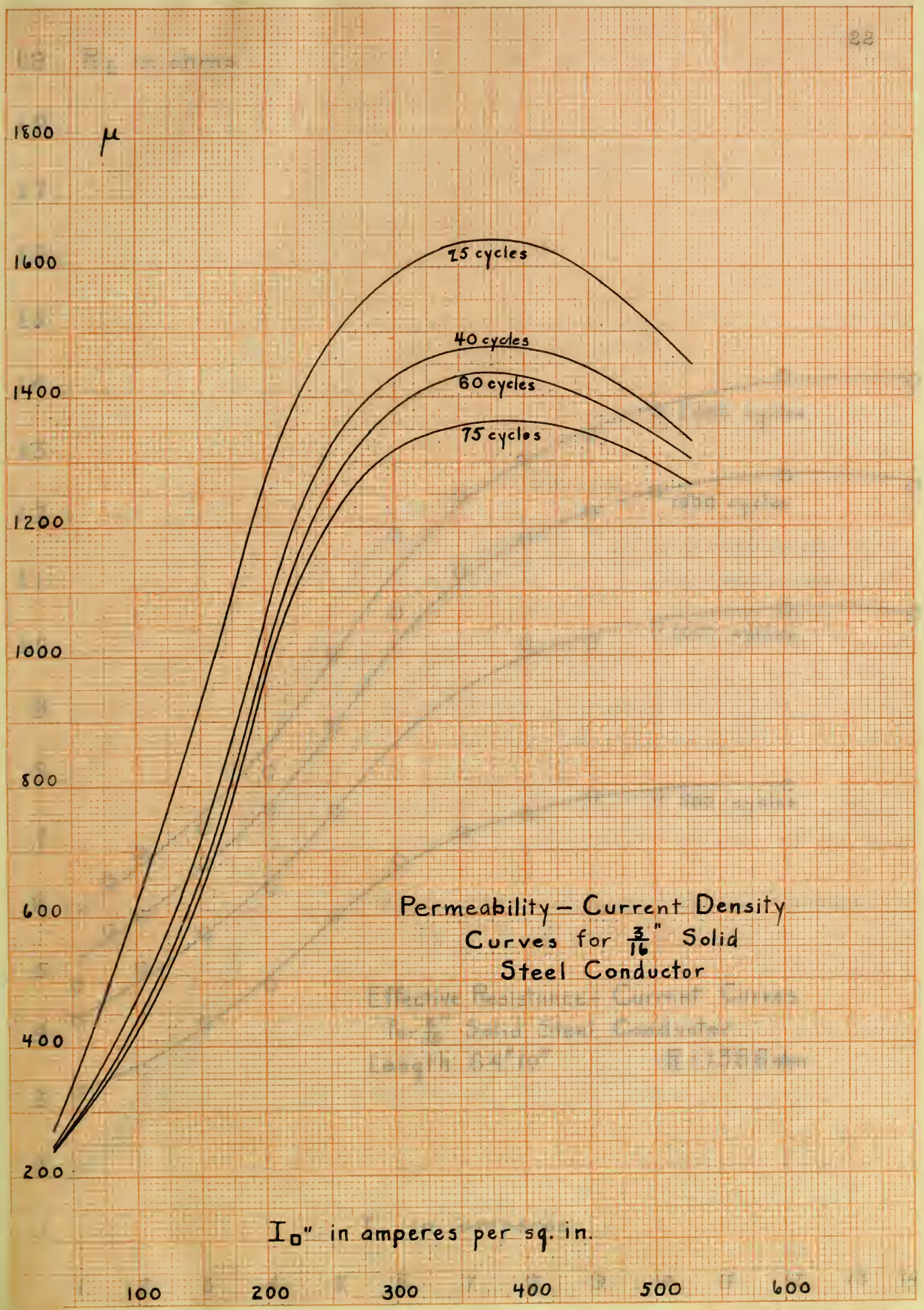
TABLE X.

X - FREQUENCY DATA FOR 3/16" SOLID STEEL CONDUCTOR.

$I_{\square''}$	I	25~ X	40~ X	60~ X	75~ X	500~ X	1000~ X	1500~ X	2000~ X
70	2.00	.0603	.0681	.074	.0842	.290	.496	.670	.771
100	2.84	.062	.073	.081	.095	.327	.538	.714	.834
150	4.26	.066	.083	.096	.121	.410	.627	.815	.965
200	5.68	.069	.096	.119	.148	.501	.728	.906	1.09
300	8.51	.089	.118	.150	.176	.552	.800	.990	1.19
400	11.34	.096	.120	.1505	.177	.543	.801	1.01	1.21
500	14.18	.095	.120	.151	.175		.778	1.015	1.216


.38 R_E in ohms
.36
.34
.32
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
75 cycles
60 cycles
40 cycles
25 cycles
Effective Resistance - Line Current Curves
for 3/16" Solid Steel Conductor
Length 64' 10"
R = .1366 ohm
I_L in amperes.
1 2 3 4 5 6 7 8 9 10 11 12 13 14

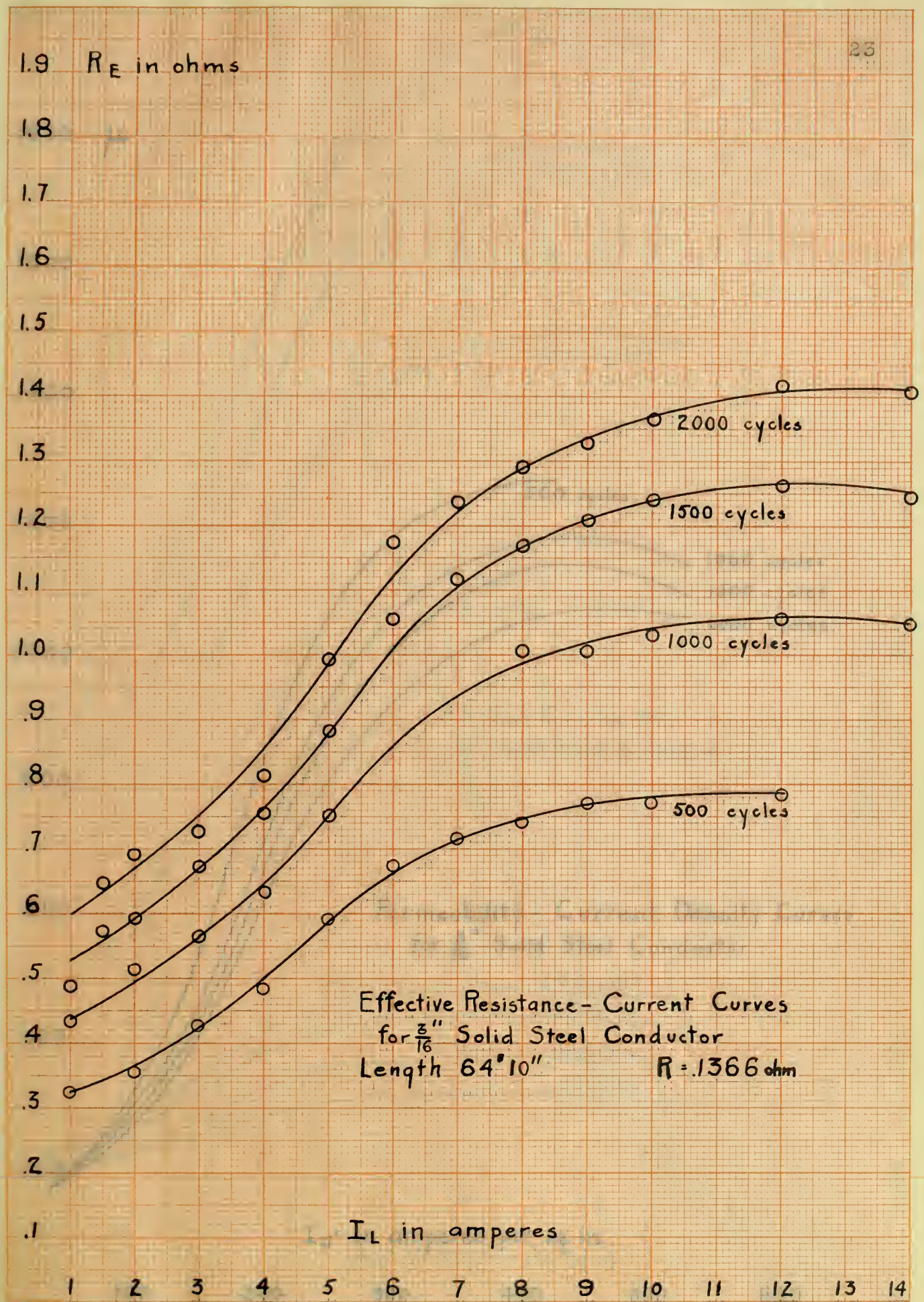
Effective Resistance - Line Current Curves
for 5/16" Solid Steel Conductor
Length 47' 10" R = .1346 ohm
R in ohms
I in amperes
75 cycles
60 cycles
40 cycles
25 cycles



Permeability – Current Density Curves for $\frac{3}{16}$" Solid Steel Conductor

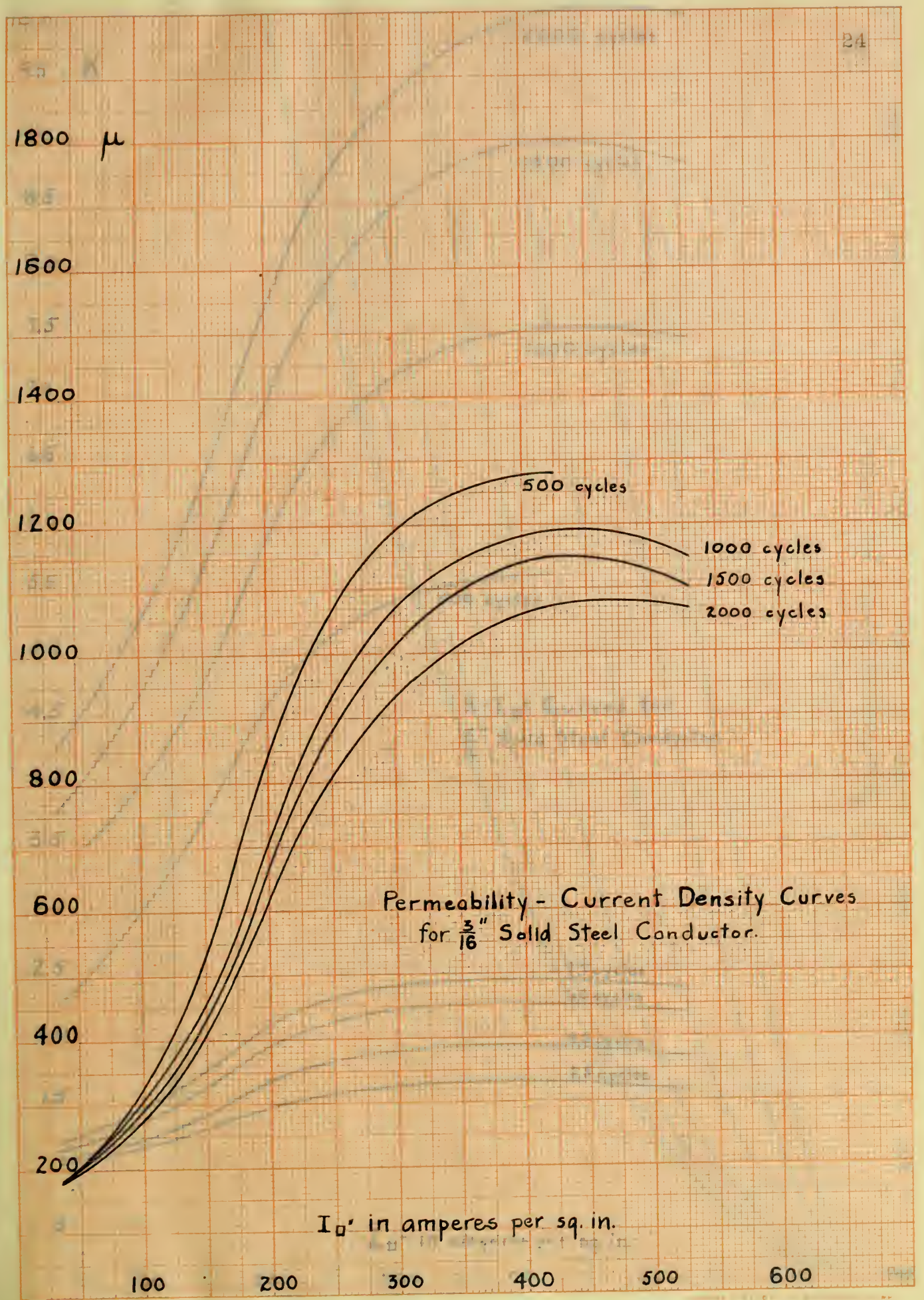
I_o" in amperes per sq. in.

Permeability – Current Density
Curves for $\frac{3}{16}$" Solid
Steel Conductor

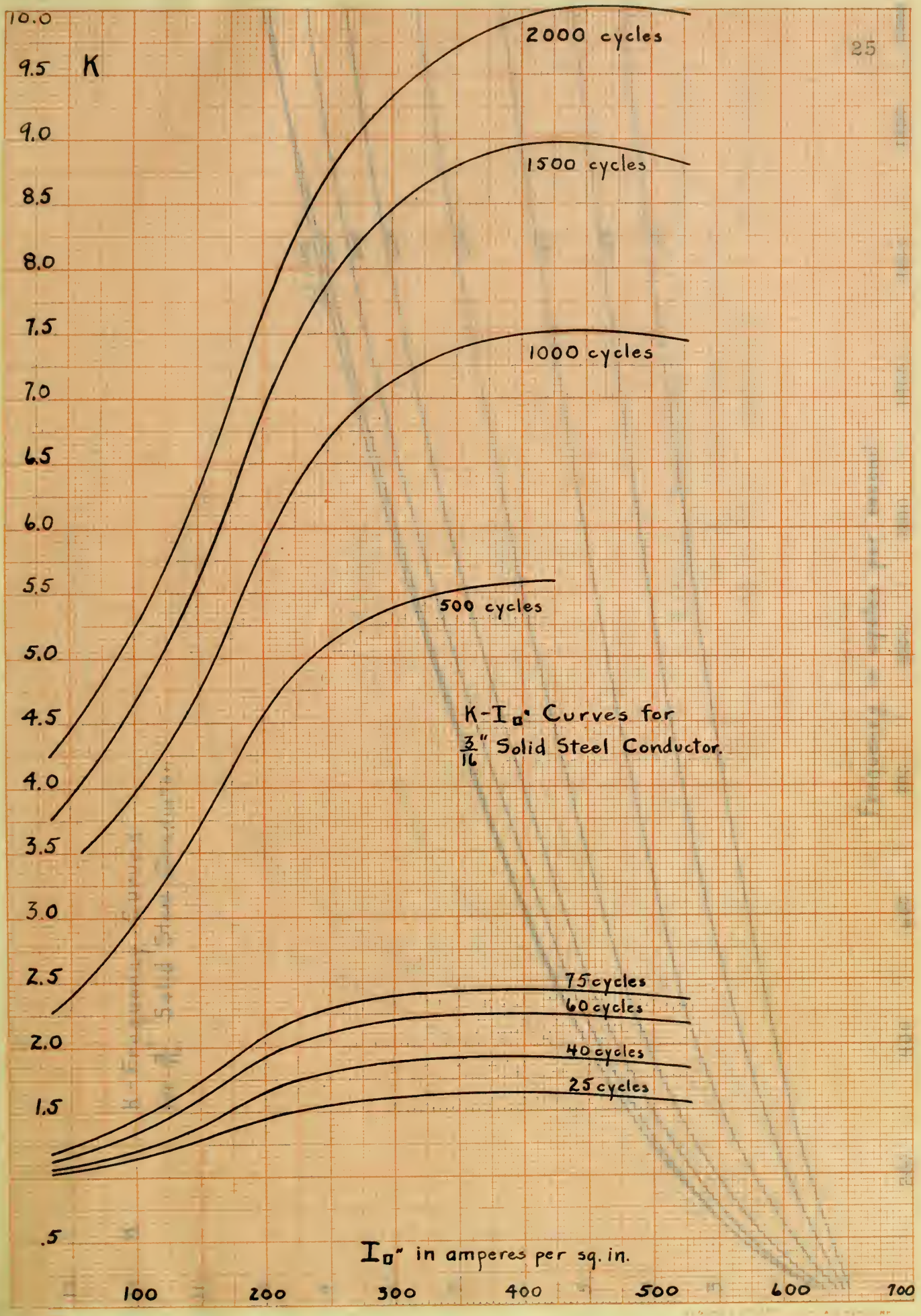


Effective Resistance – Current Curves
for $\frac{3}{16}$" Solid Steel Conductor
Length 64'10" R = .1366 ohm

2000 cycles
1500 cycles
1000 cycles
500 cycles
Effective Resistance - Current Curves
for $\frac{5}{16}$" Solid Steel Conductor
Length 64"to"
R = 1566 ohm
R_e in ohms
I_e in amperes


1800 μ
1600
1400
1200
1000
800
600
400
200
500 cycles
1000 cycles
1500 cycles
2000 cycles
Permeability - Current Density Curves
for 3/16" Solid Steel Conductor.
I□ in amperes per sq. in.
100
200
300
400
500
600

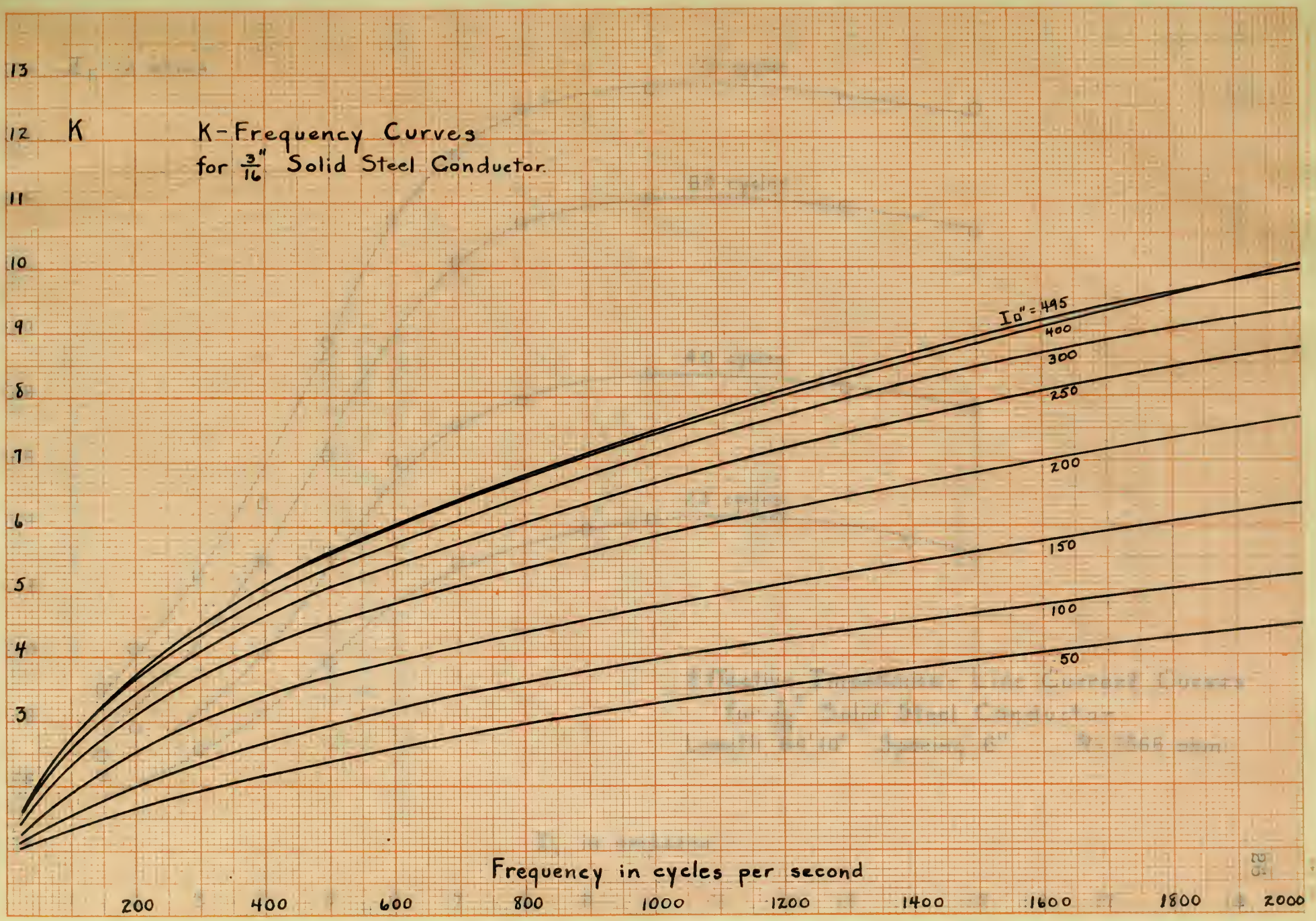
Permeability - Current Density Curves
for $\frac{5}{16}$" Solid Steel Conductor


10.0
9.5
K
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
.5
2000 cycles
1500 cycles
1000 cycles
500 cycles
75 cycles
60 cycles
40 cycles
25 cycles
K-I□" Curves for
3/16" Solid Steel Conductor.
I□" in amperes per sq. in.
100
200
300
400
500
600
700

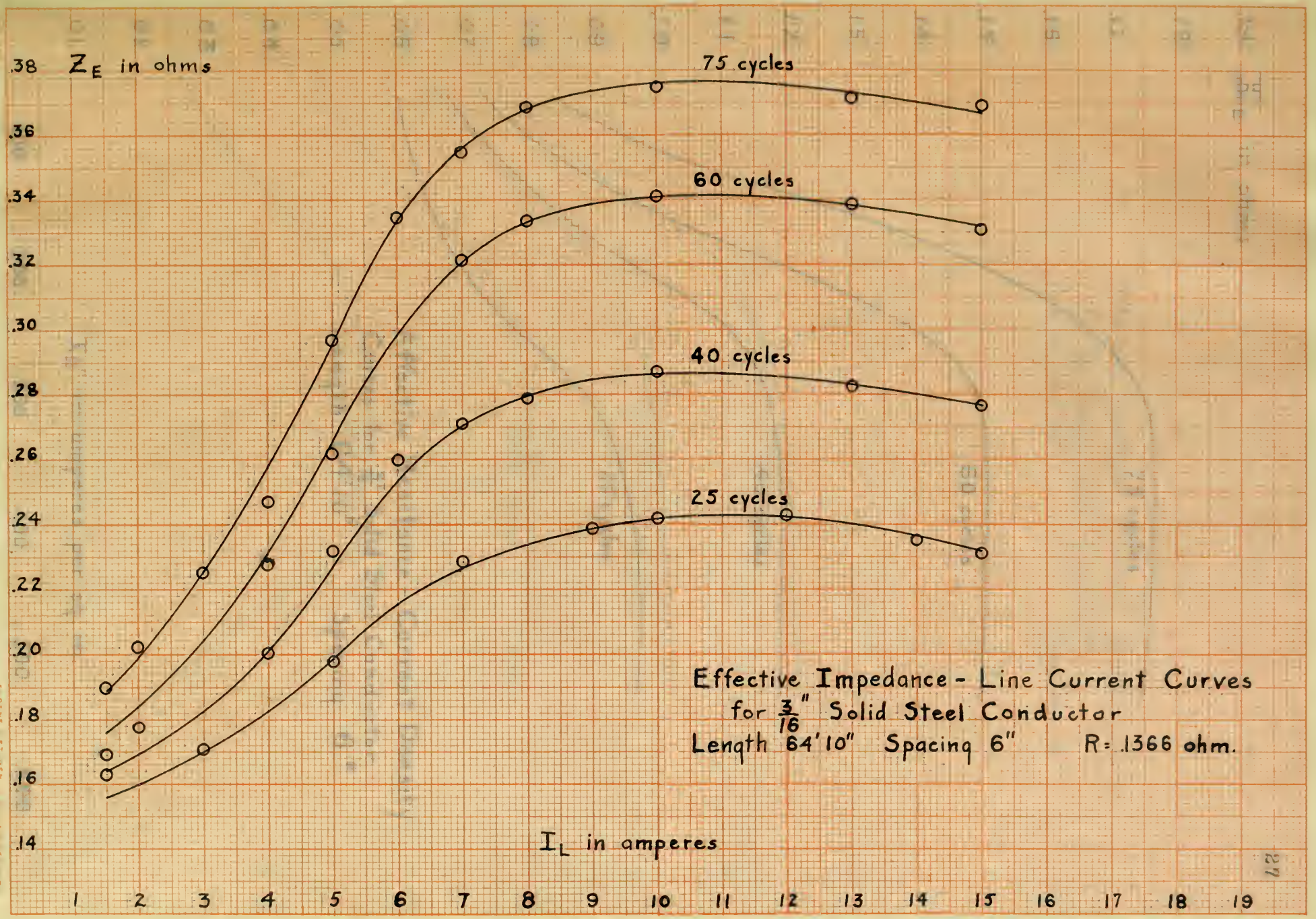
K-I_D" Curves for
$\frac{3}{16}$" Solid Steel Conductor.

K

K-Frequency Curves for $\frac{3}{16}$" Solid Steel Conductor.

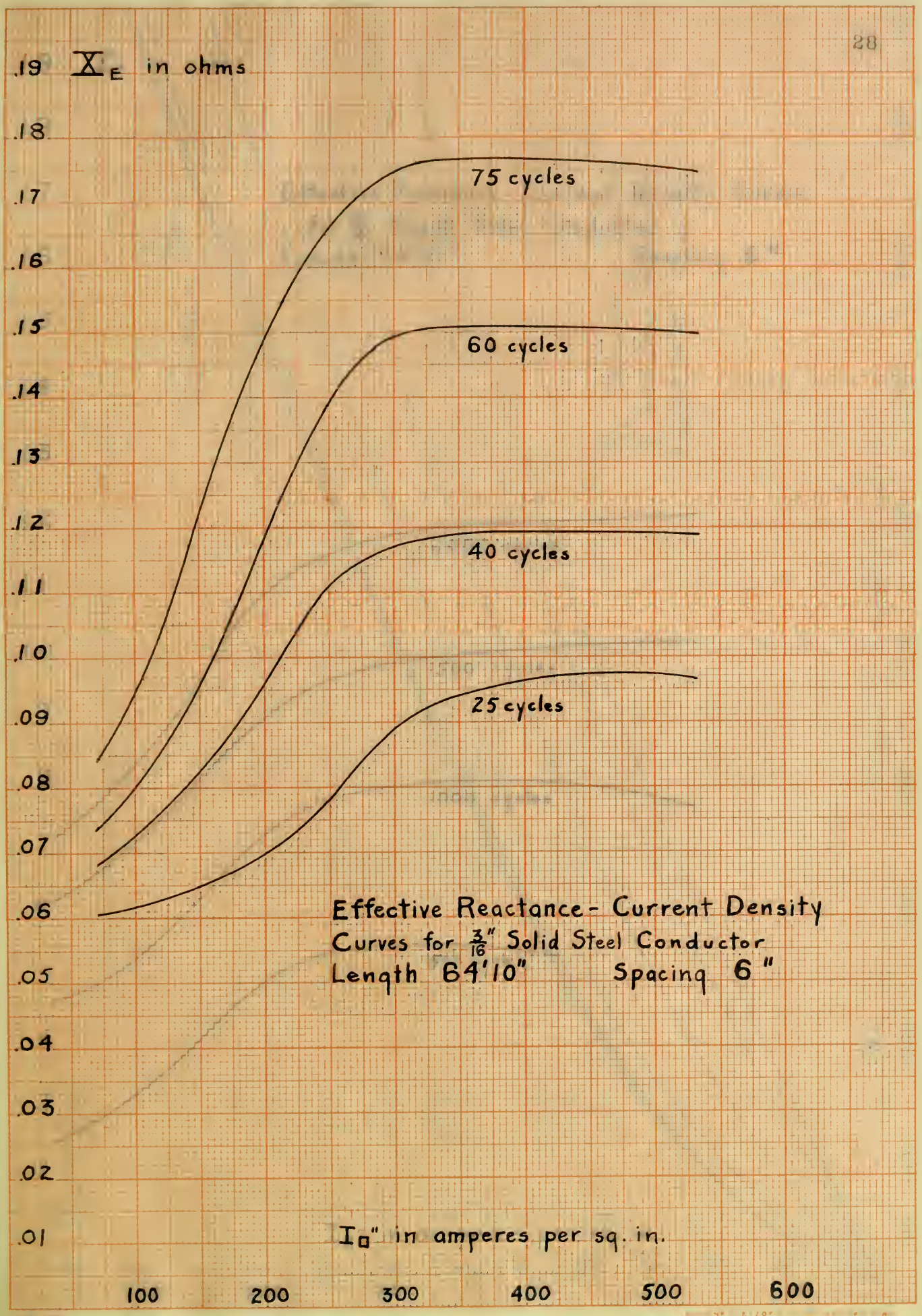


Resonance Curves
for $\frac{1}{2}$" pitch Coupling

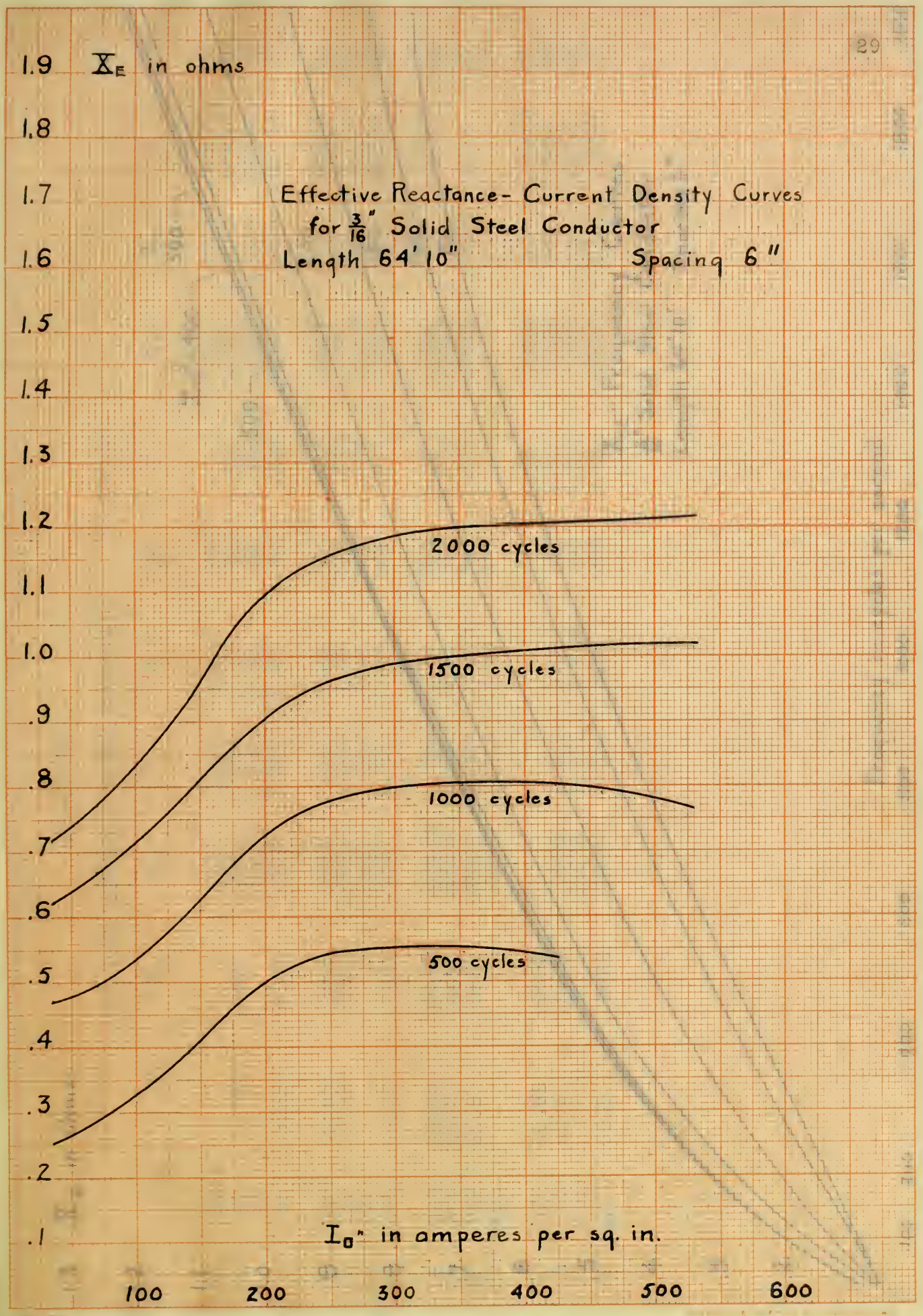


Effective Impedance - Line Current Curves
for $\frac{3}{16}$" Solid Steel Conductor
Length 64'10" Spacing 6" R = .1366 ohm.

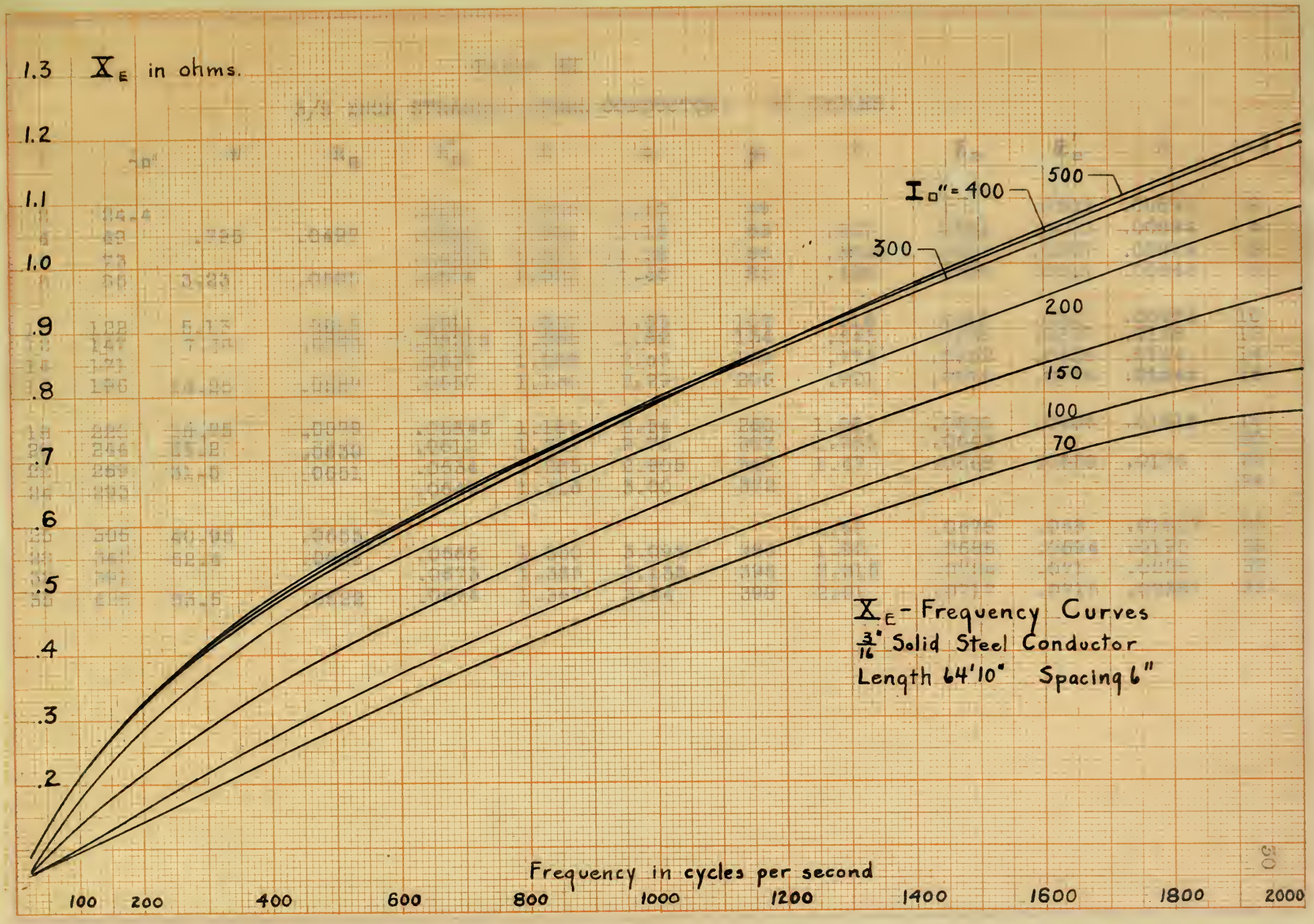
Effective Impedance - Line Current Curves
for ¾" Solid Steel Conductor
Length 84'10" Spacing 6" R = 1556 ohms.
Ohms in Line
25 cycles
50 cycles
40 cycles
25 cycles


X_E in ohms
.19
.18
.17
.16
.15
.14
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
75 cycles
60 cycles
40 cycles
25 cycles
Effective Reactance - Current Density
Curves for 3/16" Solid Steel Conductor
Length 64'10" Spacing 6"
I_a" in amperes per sq. in.
100
200
300
400
500
600

Effective Reactance - Current Density
Curves for $\frac{3}{16}$" Solid Steel Conductor
Length 84'10" Spacing 6"


1.9 X_E in ohms
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
Effective Reactance- Current Density Curves
for 3/16" Solid Steel Conductor
Length 64' 10" Spacing 6"
2000 cycles
1500 cycles
1000 cycles
500 cycles
I_a" in amperes per sq. in.
100
200
300
400
500
600

Effective Reactance- Current Density Curves
for 3/16" Solid Steel Conductor
Length 84' 10" Spacing 6"
X_e in ohms
I_d" in amperes per sq. in.
2000 cycles
1500 cycles
1000 cycles
500 cycles
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
100
200
300
400
500
600


X_E in ohms.
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
$I_{\square}$" = 400
500
300
200
150
100
70
X_E - Frequency Curves
$\frac{3}{16}$" Solid Steel Conductor
Length 64'10" Spacing 6"
Frequency in cycles per second
100
200
400
600
800
1000
1200
1400
1600
1800
2000

Frequency Characteristic Curves
Reactance
Solid Wire Conductor
"
Spacing
10 Mils

TABLE XI.

3/8 INCH STRANDED STEEL CONDUCTOR. 25 CYCLES.

I	$I_{0''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
2	24.4			.0494	1.002	1.10	49			.0502	.00892	2
4	49	.795	.0497	.0495	1.004	1.15	53	.201	.0503	.0503	.00894	4
6	73			.04985	1.011	1.28	66	.3045	.0507	.0507	.00924	6
8	85	3.23	.0505	.0504	1.022	1.45	85	.406	.0508	.0511	.00843	8
10	122	5.13	.0513	.0511	1.037	1.63	107	.516	.0516	.052	.00953	10
12	147	7.56	.0525	.05215	1.058	1.82	134	.643	.0536	.0537	.0128	12
14	171			.0537	1.089	2.05	169	.773	.0552	.0556	.0144	14
16	196	14.25	.0557	.0557	1.130	2.29	206	.921	.0575	.0578	.01544	16
18	220	18.75	.0579	.05845	1.185	2.54	260	1.08	.0600	.0606	.01618	18
20	244	25.2	.0630	.0612	1.241	2.76	307	1.285	.0643			20
22	269	31.5	.0651	.0634	1.286	2.905	340	1.47	.0668	.0658	.0176	22
24	293			.0649	1.316	3.00	372					24
25	306	40.95	.0655					1.69	.0676	.068	.01827	25
28	342	52.4	.0668	.0666	1.350	3.095	386	1.92	.0686	.0694	.0195	28
32	391			.0673	1.365	3.135	396	2.315	.0724	.071	.0226	32
35	428	83.5	.0622	.0674	1.367	3.14	398	2.51	.0717	.0715	.02387	35

TABLE XII.

3/8 INCH STRANDED STEEL CONDUCTOR. 40 Cycles.

I	$I_{0''}$	W	R_E	R'_E	K	q	μ
2	24.4			.0495	1.004	1.15	33
3	36.7	.449	.0498				
4	49			.050	1.014	1.35	46
6	73			.0507	1.028	1.53	59
8	85	3.31	.0517	.0516	1.047	1.73	75
10	122	5.20	.052	.0528	1.070	1.92	93
12	147	7.86	.0546	.0543	1.101	2.13	114
14	171	11.0	.0561	.0570	1.156	2.42	147
16	196			.0614	1.245	2.775	194
18	220	21.7	.0670	.0660	1.339	3.065	236
20	244	27.7	.0693	.0695	1.409	3.265	268

TABLE XIII.

3/8 INCH STRANDED STEEL CONDUCTOR. 60 CYCLES.

I	$I_{\square''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
2	24.4			.04985	1.011	1.300	28	.1075	.0537	.0538	.02036	2
3	36.7	.453	.0503									3
4	49	.805	.0503	.0506	1.027	1.515	38	.215	.0538	.0547	.0208	4
6	73			.0516	1.047	1.73	50	.341	.0568	.0559	.0215	6
8	85	3.42	.0534	.0529	1.073	1.94	63	.460	.0575	.0579	.0235	8
10	122	5.43	.0543	.05455	1.106	2.16	78	.589	.0589	.0604	.0259	10
12	147	8.11	.0563	.0570	1.156	2.475	103			.0634	.02775	12
14	171	11.45	.0584	.0618	1.254	2.81	132			.0688	.0302	14
16	196			.0692	1.404	3.24	176	1.29	.0806	.0763	.03215	16
18	220	25.45	.0785	.0765	1.552	3.65	223	1.526	.0848	.0840	.0347	18
20	244	32.8	.0820	.0818	1.659	3.945	261	1.782	.0891			20
22	269	41.45	.0856	.08525	1.729	4.117	284	2.052	.0933	.0937	.0389	22
25	306	55.5	.0888	.0891	1.807	4.345	316	2.44	.0976	.0987	.04245	25
28	342	73.0	.0931	.0917	1.860	4.495	339	2.855	.102	.1017	.0440	28
32	391	96.6	..0943	.0937	1.900	4.61	356	3.33	.104	.1039	.0449	32
35	428	116.2	.0949	.0946	1.918	4.655	363	3.66	.1045	.1044	.0442	35

TABLE XIV.

3/8 INCH STRANDED STEEL CONDUCTOR. 90 CYCLES.

I	$I_{\square}$'	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
2	24.4	.202	.0505	.0505	1.0245	1.485	25	.116	.0580	.0575	.0275	2
3	36.7	.47	.0522					.174	.0580			3
4	49			.05135	1.042	1.68	32			.0593	.02965	4
6	73			.0535	1.085	2.03	46			.0619	.0311	6
8	85	3.61	.0564	.0557	1.130	2,29	59	.518	.0647	.065	.0335	8
10	122	5.87	.0587	.0585	1.186	2.545	73	.686	.0686	.0686	.0358	10
12	147	8.96	.0622	.0624	1.265	2.85	91	.879	.0732	.0732	.03825	12
14	171	13.45	.0686	.0699	1.418	3.28	121	1.144	.0817	.0806	.0401	14
16	196	19.25	.0752	.0805	1.632	3.865	168	1.425	.0891	.0906	.0416	16
18	220	29.25	.0903	.0889	1.803	4.34	212	1.826	.1014	.0997	.0451	18
20	244	37.95	.0949	.0947	1.920	4.66	244	2.127	.1064			20
22	269	47.5	.0982	.0988	2.004	4.90	270	2.422	.1100	.1109	.0504	22
25	306	63.8	.1020	.1031	2.092	5.14	297	2.908	.1163	.1166	.0544	25
28	342			.1068	2.166	5.36	323	3.423	.1222	.1204	.0556	28
32	391	112.8	.1101	.1099	2.23	5.53	344	3.968	.124	.1239	.0572	32
35	428	135.4	.1105	.1106	2.243	5.56	348	4.404	.1258	.1255	.0592	35

TABLE XV.

3/8 INCH STRANDED STEEL CABLE. 1000 CYCLES.

I	I_0'	W	R_E	R_E'	K	q	μ	E	Z_E	Z_E'	X	I
1	12.2	.10	.100									1
1.5	18.3	.232	.1031									1.5
2	24.4	.417	.1042	.1055	2.18	5.39	29	.447	.2235	.2275	.2016	2
3	36.7	1.00	.1111	.1093				.712	.2373	.2322	.2049	3
4	49	1.88	.1175	.114	2.31	5.76	33.5	.964	.241	.2378	.2088	4
5	61			.1181				1.236	.2472	.243	.2123	5
6	73			.1228	2.49	6.26	39	1.505	.2509	.2485	.2161	6
8	85	8.48	.1325	.132	2.68	6.81	47	2.092	.2616	.2605	.2246	8
10	122	13.9	.139	.142	2.88	7.38	55	2.778	.2778	.276	.2367	10
12	147	21.6	.150	.152	3.08	7.94	63	3.434	.2861	.2895	.2463	12
14	171	31.7	.1617	.163	3.307	8.59	74	4.25	.3036	.303	.2554	14
17	208	55.4	.1917	.1808	3.668	9.61	93	5.50	.3235	.322	.2664	17
20	244	80	.200	.1995	4.048	10.67	115	6.79	.3395	.338	.2728	20
22	269	102.9	.2126	.2108	4.276	11.32	129	7.68	.349	.348	.277	22
25	306	140.3	.2245	.224	4.545	12.09	147	8.98	.359	.360	.2819	25
28	342	182.6	.2330	.2335	4.738	12.62	160	10.13	.3618	.368	.2844	28
30	367	214.5	.2383	.238	4.83	12.89	167	11.15	.3718	.371	.2846	30

TABLE XVI.

3/8 INCH STRANDED STEEL CONDUCTOR. 2000 CYCLES.

I	$I_{0''}$	W	R_E	R'_E	K	q	μ	E	Z_E	Z'_E	X	I
1.5	18.3	.324	.144					.570	.380			1.5
2	24.4	.582	.1455	.146	2.962	7.61	29			.3858	.3572	2
3	36.7			.1508				1.196	.3986	.3925	.3623	3
4	49	2.52	.1575	.1555	3.155	8.16	33.5	1.568	.392	.3995	.3676	4
5	61			.160						.4067	.374	5
6	73			.164	3.328	8.64	38	2.494	.4158	.4145	.3808	6
8	85	11.3	.1789	.1755	3.56	9.30	43	3.443	.4304	.430	.3926	8
10	122	18.5	.185	.1855	3.763	9.88	49	4.58	.458	.4465	.406	10
12	147	28.4	.1972	.1965	3.986	10.51	55	5.64	.470	.4625	.4187	12
14	171			.207	4.20	11.11	62	6.63	.4735	.4755	.428	14
17	208	64.5	.2231	.223	4.524	12.02	73	8.56	.5034	.498	.4452	17
20	244	95.5	.2387	.242	4.910	13.11	87	10.29	.5145	.517	.4568	20
22	269	124.2	.2566	.2543	5.16	13.82	96	11.68	.531	.529	.4638	22
25	306	170	.272	.2725	5.53	14.87	111	13.51	.5408	.544	.4707	25
28	342	226.7	.2892	.287	5.82	15.69	124	15.53	.5548	.554	.4738	28
30	367	265	.2945	.295	5.98	16.16	131	16.76	.5586	.5575	.473	30

TABLE XVII.

I - K DATA FOR 3/8" STRANDED STEEL CONDUCTOR

I	$I_{\square'}$	25~ K	60~ K	90~ K	1000~ K	2000~ K	I
2	24.4	1.002	1.011	1.0245	2.18	2.962	2
4	49	1.004	1.027	1.042	2.31	3.155	4
6	73	1.011	1.047	1.085	2.49	3.328	6
8	85	1.022	1.073	1.130	2.68	3.56	8
10	122	1.037	1.106	1.186	2.88	3.763	10
12	147	1.058	1.156	1.265	3.08	3.986	12
14	171	1.089	1.254	1.418	3.307	4.20	14
17	208	1.158	1.482	1.728	3.668	4.524	17
20	244	1.241	1.659	1.920	4.048	4.910	20
22	269	1.286	1.729	2.004	4.276	5.16	22
25	306	1.325	1.807	2.092	4.545	5.53	25
28	342	1.350	1.860	2.166	4.738	5.82	28
30	367				4.83	5.98	30
32	391	1.365	1.900	2.23			32
35	428	1.367	1.918	2.243			35

TABLE XVIII.

K - FREQUENCY DATA FOR 3/8" STRANDED STEEL CONDUCTOR.

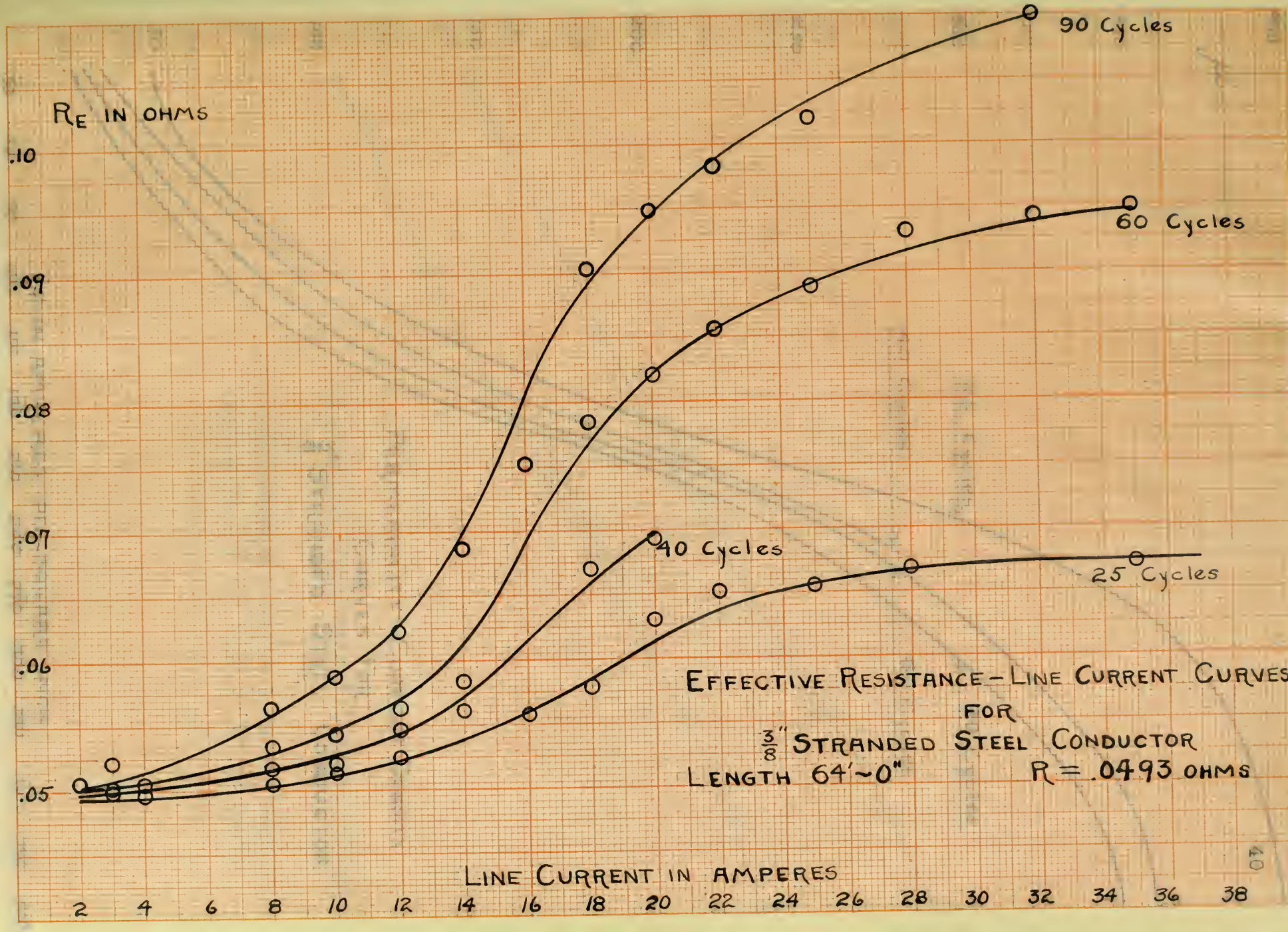
I_d"	I	25~ K	60~ K	90~ K	1000~ K	2000~ K	I_d"
50	4.09	1.006	1.034	1.052	2.33	3.155	50
100	8.18	1.022	1.078	1.135	2.695	3.56	100
150	12.27	1.058	1.174	1.288	3.12	4.01	150
200	16.36	1.14	1.438	1.67	3.59	4.455	200
250	20.46	1.258	1.675	1.944	4.12	4.985	250
300	24.56	1.32	1.80	2.084	4.515	5.49+	300
350	28.64	1.354	1.866	2.18	4.77-	5.89-	350
400	32.72	1.366	1.907	2.233			400

TABLE XIX.

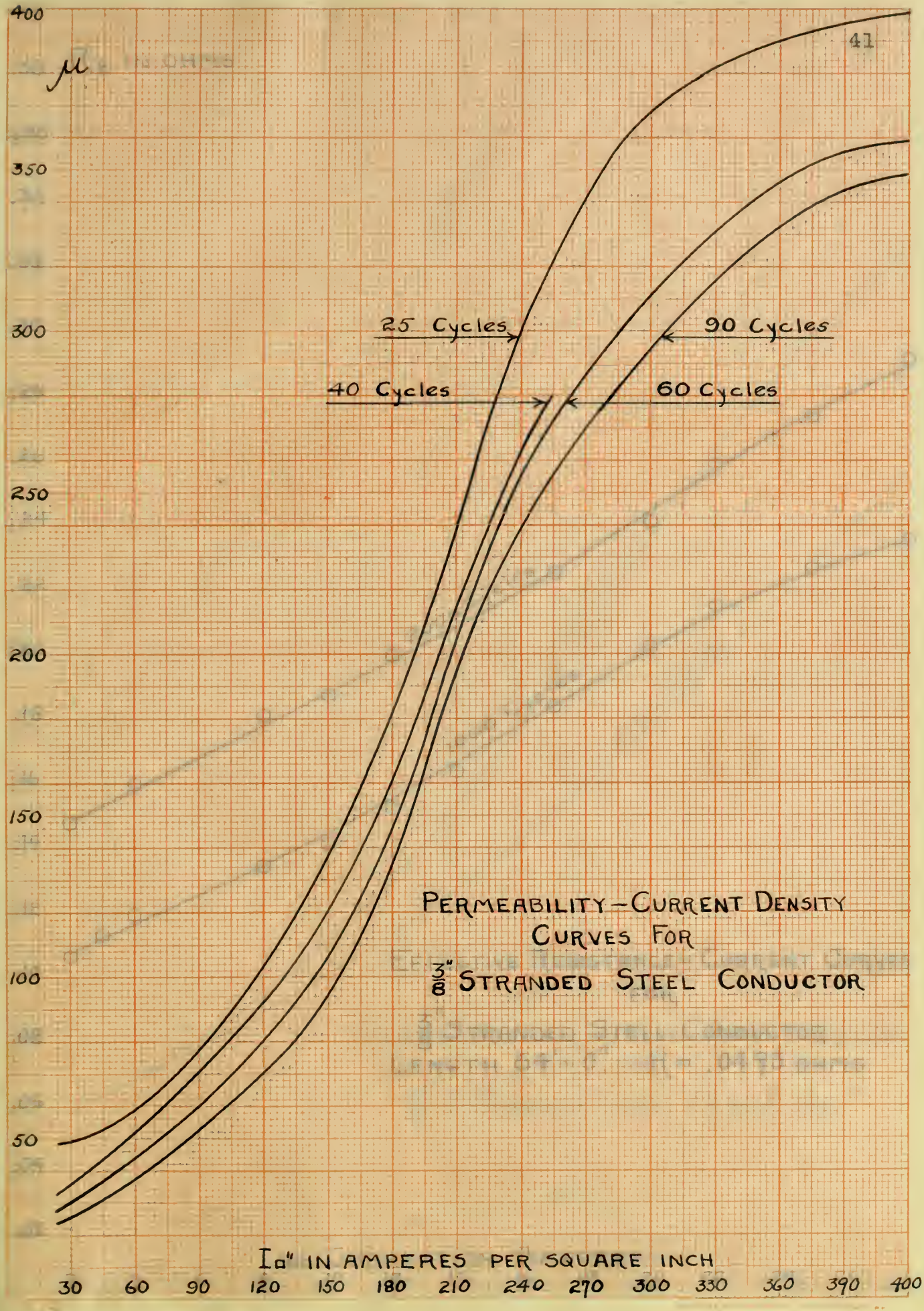
X - FREQUENCY DATA FOR 3/8" STRANDED STEEL CONDUCTOR.

$I_{\square}$"	I	25~ X	60~ X	90~ X	1000~ X	2000~ X
50	4.09	.0090	.0205	.0294	.209	.369
100	8.18	.0103	.0237	.0335	.226	.395
150	12.27	.0126	.0281	.0383	.247	.419
200	16.36	.0147	.0326	.0432	.263	.440
250	20.46	.0168	.0372	.0481	.274	.459
300	24.56	.0190	.0413	.0531	.281	.470
350	28.64	.0207	.0455	.0564	.284	.474
400	32.72	.0253	.0463	.0584		



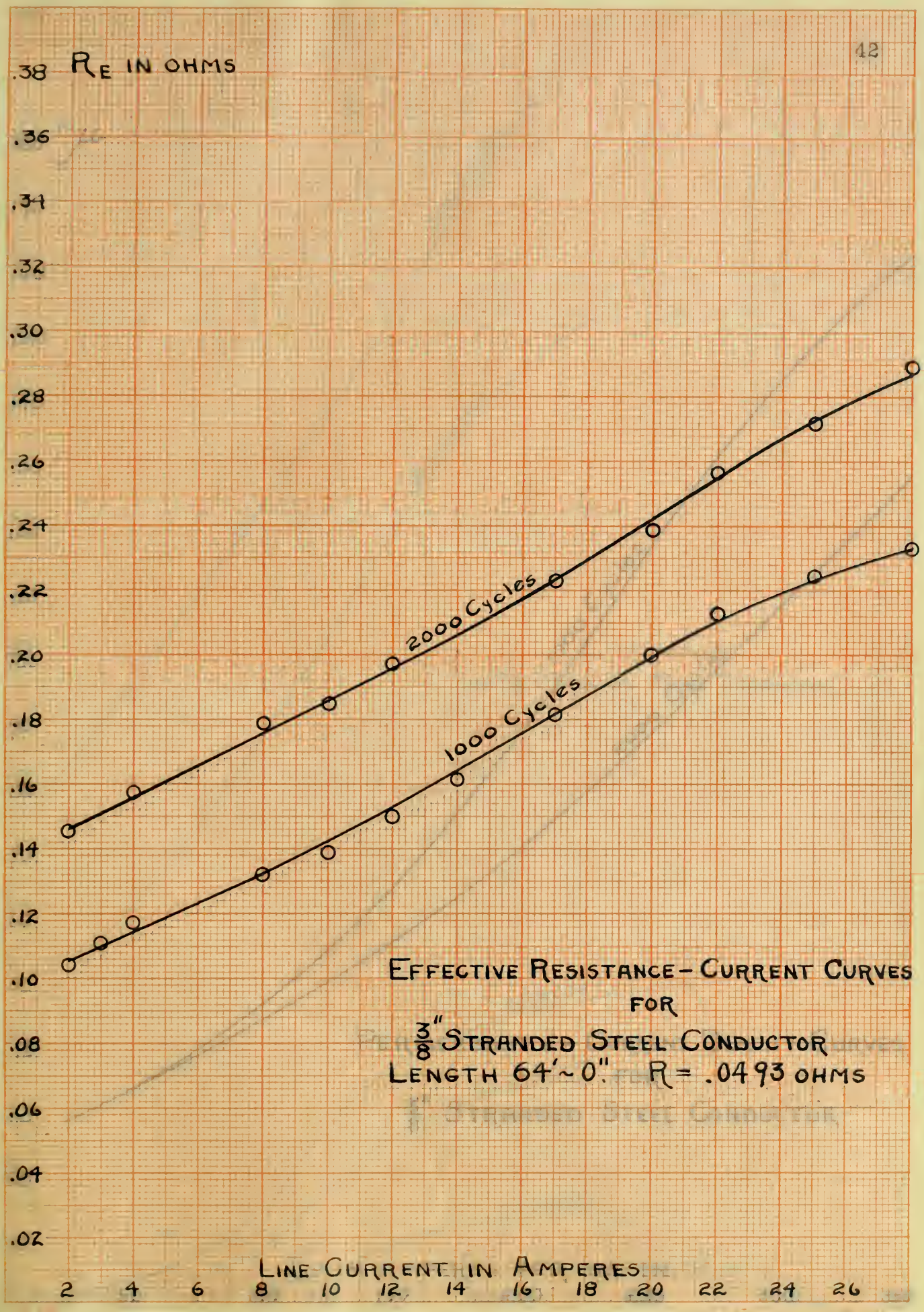

R_E IN OHMS
.10
.09
.08
.07
.06
.05
90 Cycles
60 Cycles
40 Cycles
25 Cycles
EFFECTIVE RESISTANCE – LINE CURRENT CURVES
FOR
3/8" STRANDED STEEL CONDUCTOR
LENGTH 64'-0"
R = .0493 OHMS
LINE CURRENT IN AMPERES
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38

EFFECTIVE RESISTANCE - LINE CURRENT CURVES
FOR
¾" STANDARD STEEL CONDUCTOR
LENGTH 2.4 "D" R = .0242 OHMS
LINE CURRENT IN AMPERES
25 CYCLES
50 CYCLES
60 CYCLES
EFFECTIVE RESISTANCE IN OHMS



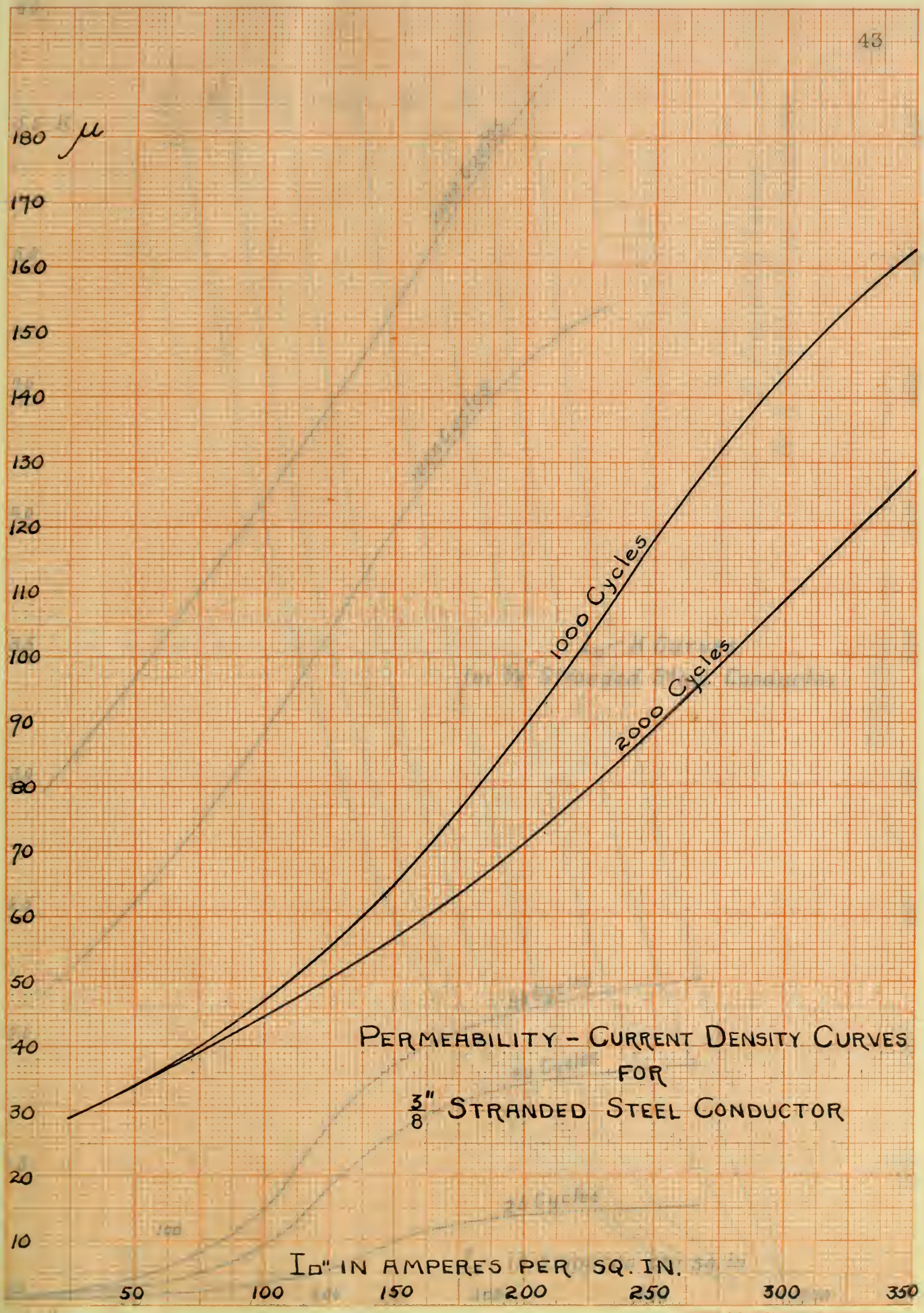
PERMEABILITY – CURRENT DENSITY CURVES FOR $\frac{3}{8}$" STRANDED STEEL CONDUCTOR

PERMEABILITY—CURRENT DENSITY
CURVES FOR
$\frac{3}{8}$" STRANDED STEEL CONDUCTOR



EFFECTIVE RESISTANCE – CURRENT CURVES
FOR
$\frac{3}{8}$" STRANDED STEEL CONDUCTOR
LENGTH 64'~0" R = .0493 OHMS

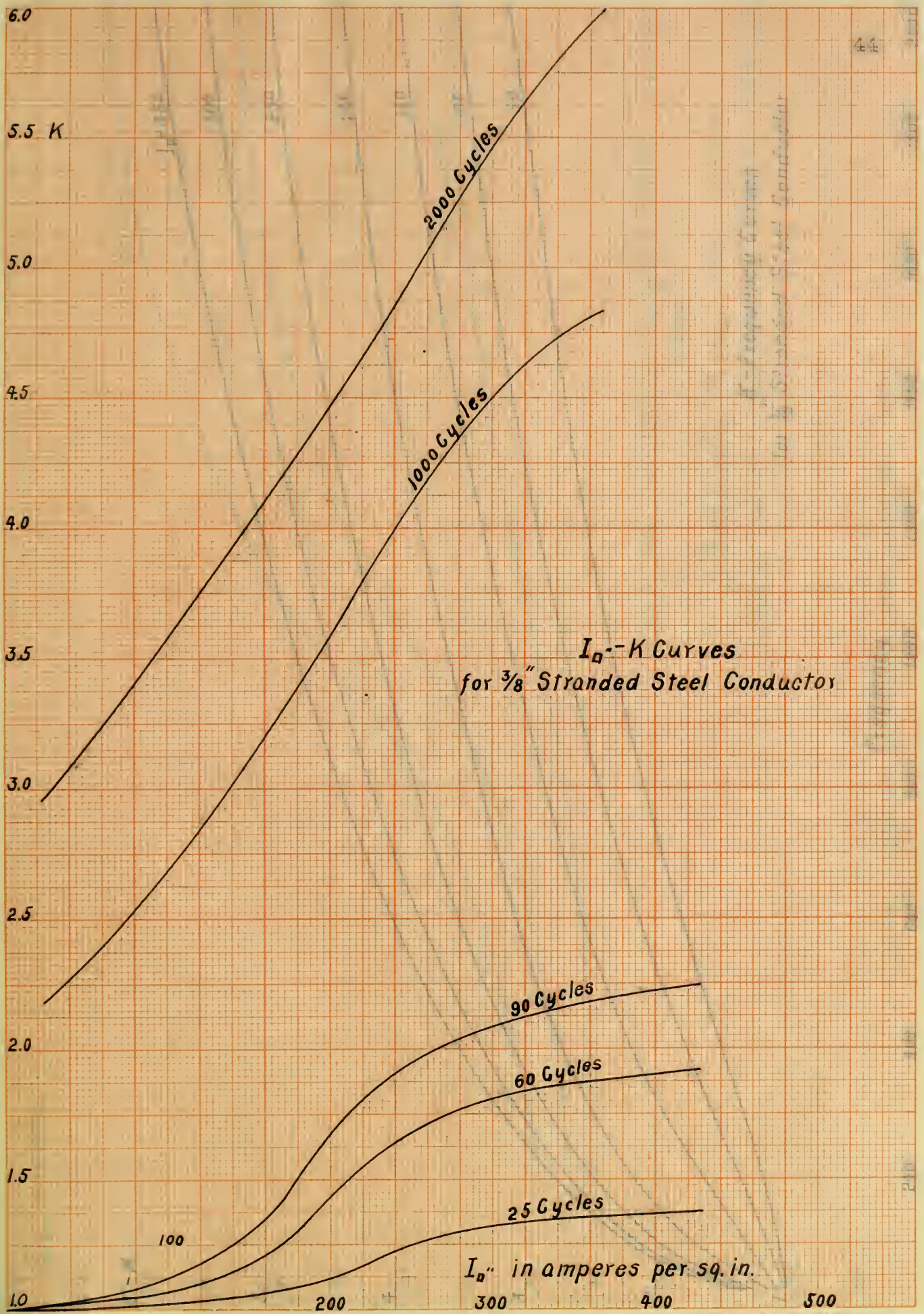
Effective Resistance – Current Curves
for
⅜" Stranded Steel Conductor
Length 61'-0" R = .0493 ohms
5000 Cycles
1000 Cycles
R in ohms
Line Current in Amperes



PERMEABILITY – CURRENT DENSITY CURVES
FOR
$\frac{3}{8}$" STRANDED STEEL CONDUCTOR

PERMEABILITY - CURRENT DENSITY CURVES
FOR
$\frac{3}{8}$" 3 STRANDED STEEL CONDUCTOR

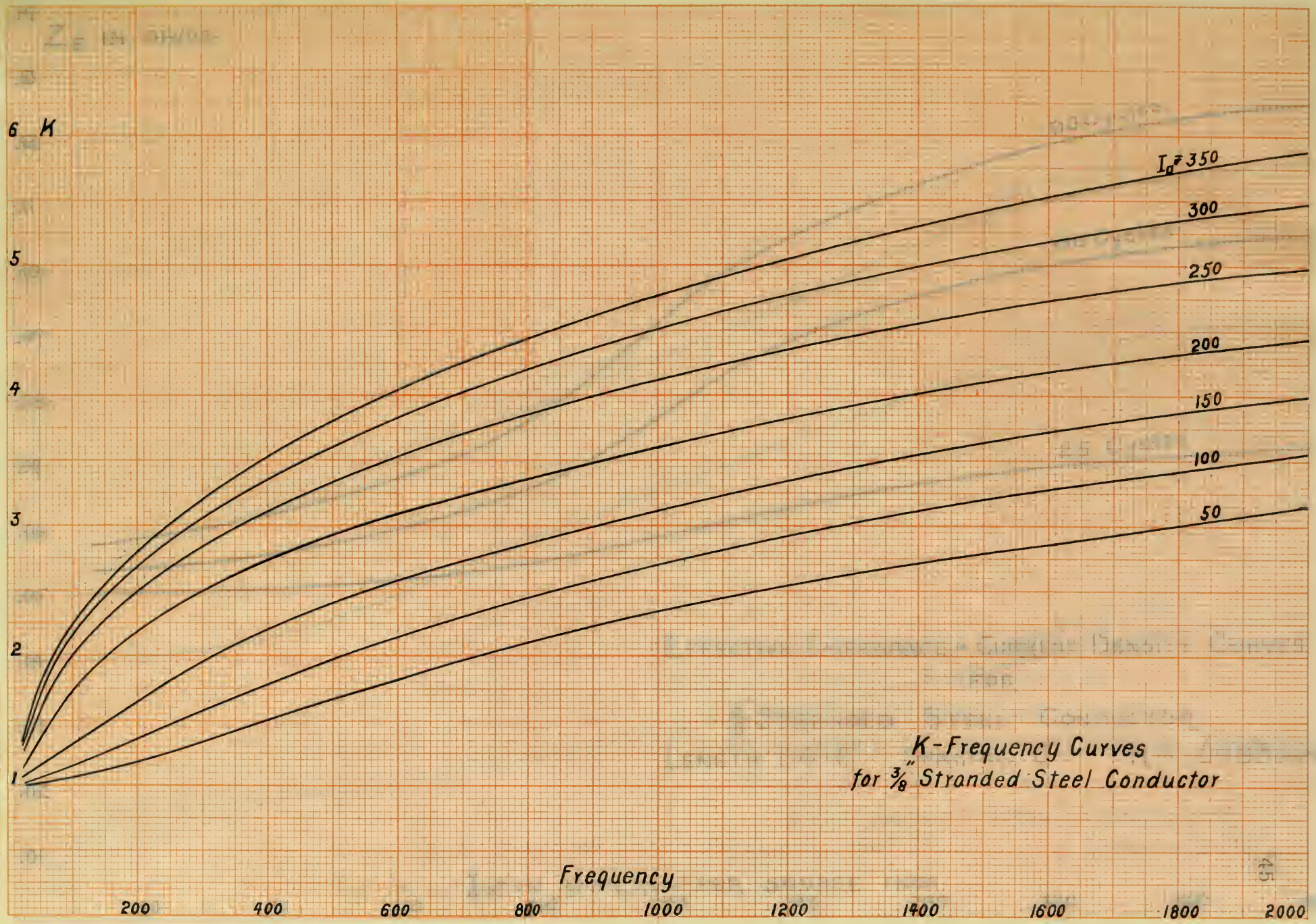
I_D IN AMPERES PER SQ. IN.


6.0
5.5 K
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
2000 Cycles
1000 Cycles
90 Cycles
60 Cycles
25 Cycles
100
200
300
400
500
I_n - K Curves
for 3/8" Stranded Steel Conductor
I_n" in amperes per sq. in.

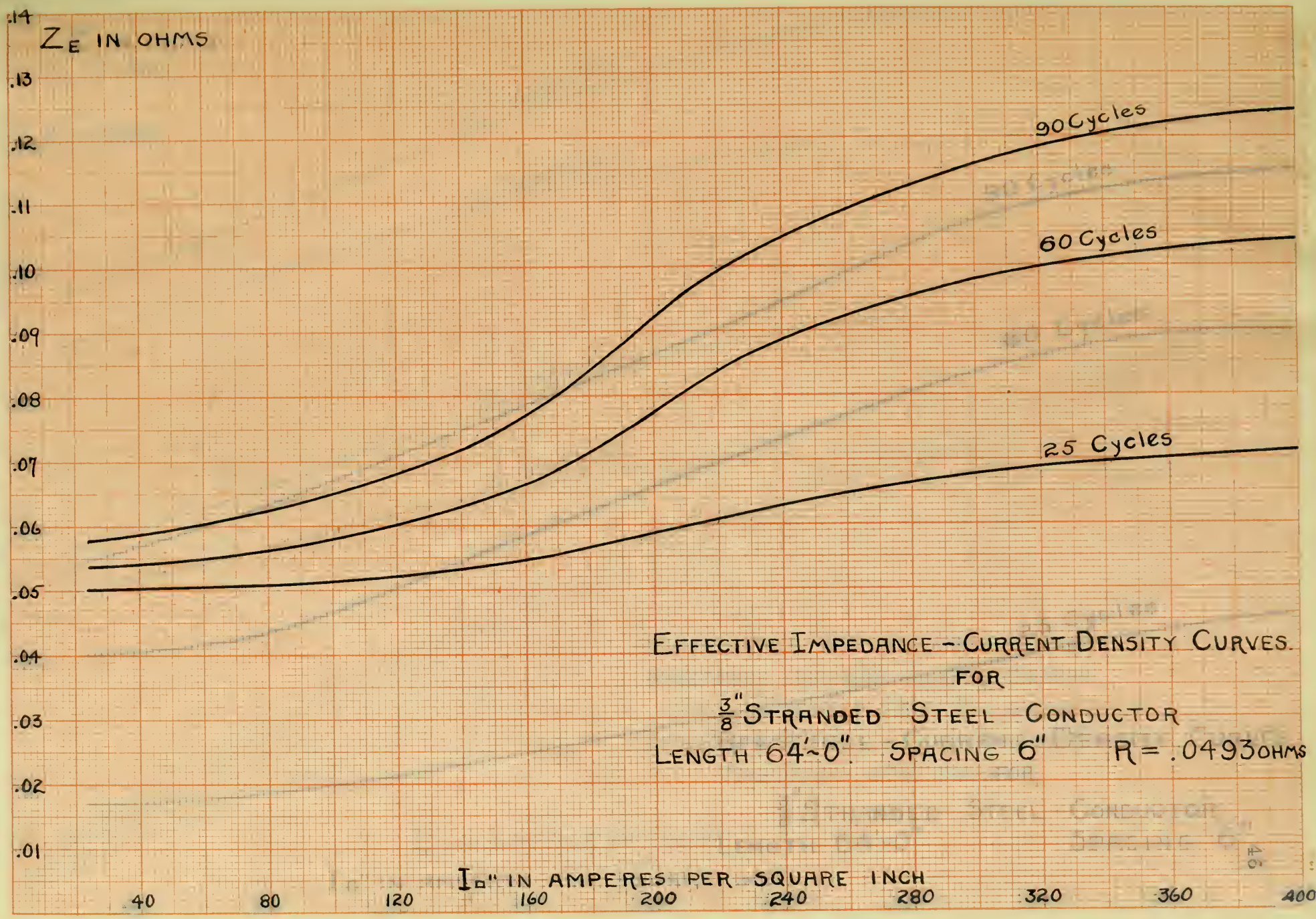
I_a - K Curves

for ⅜" Stranded Steel Conductor

I_a - In amperes per sq. in.

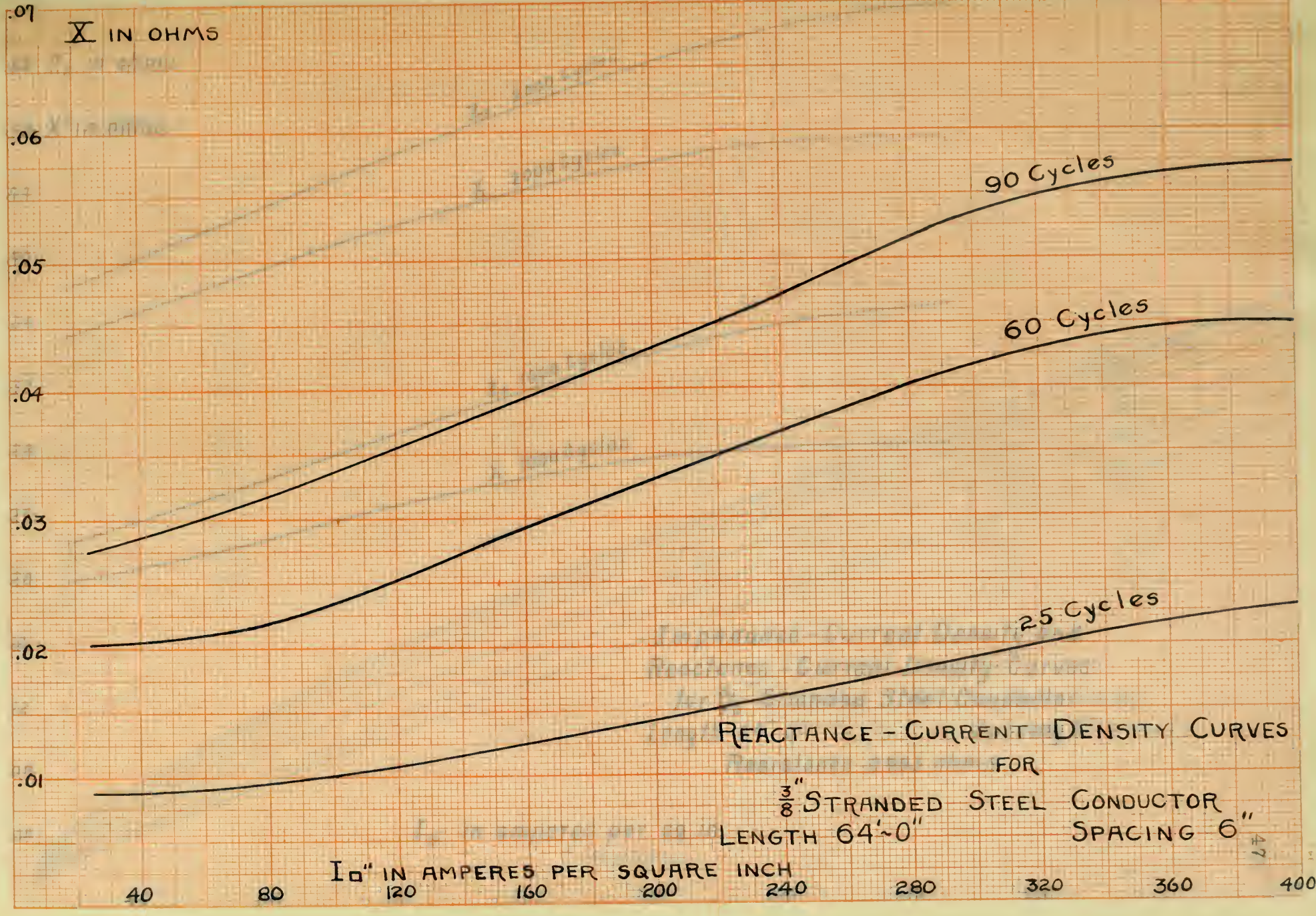

6 K
5
4
3
2
1
I_0 = 350
300
250
200
150
100
50
K-Frequency Curves
for 3/8" Stranded Steel Conductor
Frequency
200
400
600
800
1000
1200
1400
1600
1800
2000

Frequency
M-Frequency Curves
for 1½ Straight Slot Conductor
50
100
150
200
250
300
I_p=320
M
2
4
6
500
1000
1500
2000
2500
3000
3500
4000
4500
5000



EFFECTIVE IMPEDANCE - CURRENT DENSITY CURVES.
FOR
$\frac{3}{8}$" STRANDED STEEL CONDUCTOR
LENGTH 64'-0" SPACING 6" R = .0493 OHMS

ELECTRIC IMPEDANCE - CURRENT DENSITY CURVES
Length 0.40", Square Section
Rotating Test Specimen
$\frac{3}{8}$" dia. H = 0.4320 ohms
60 Cycles
25 Cycles
Ohms in Σ
Area Square Inch

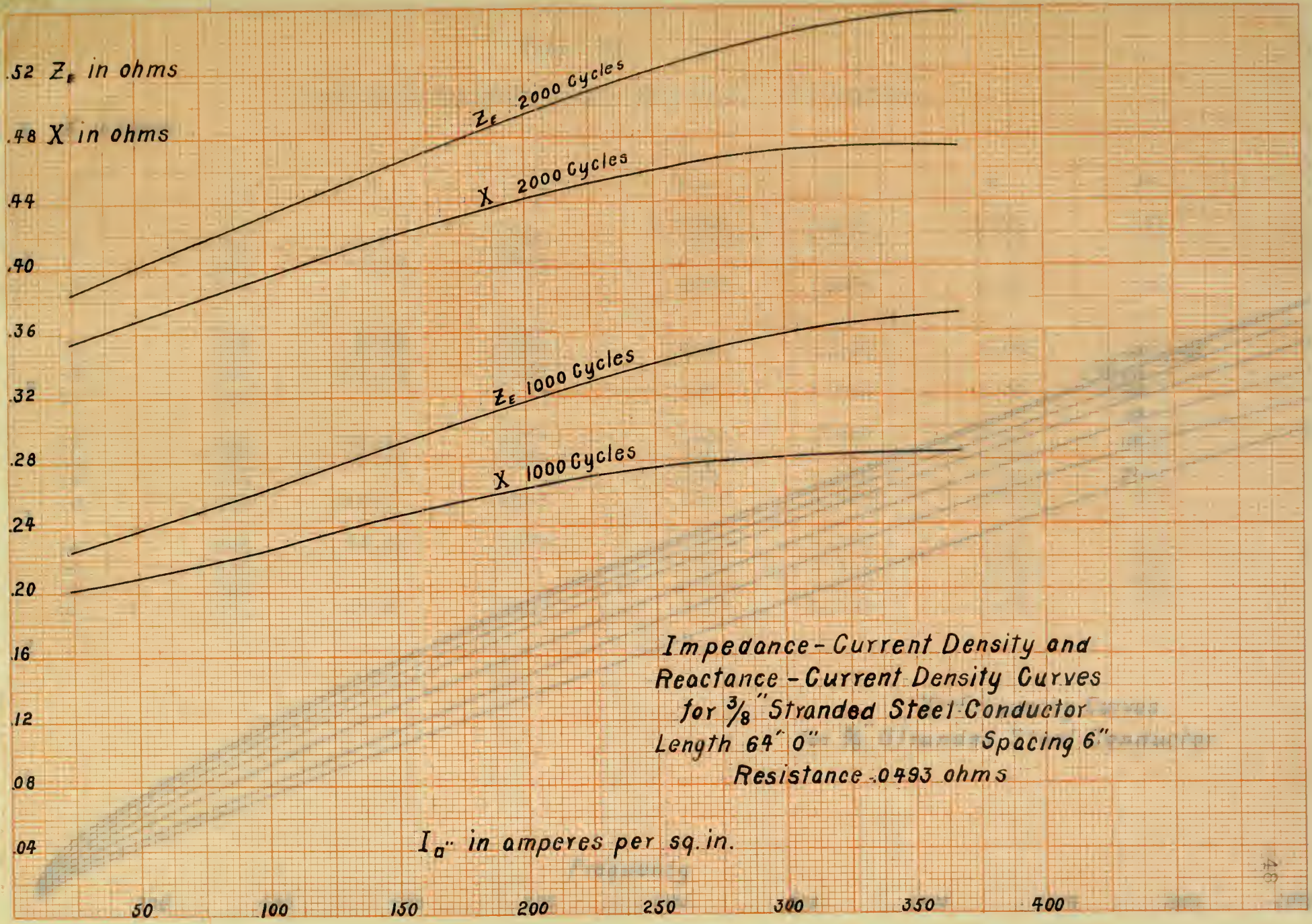


REACTANCE - CURRENT DENSITY CURVES
FOR
$\frac{3}{8}$" STRANDED STEEL CONDUCTOR
LENGTH 64'-0" SPACING 6"

Fig. 1. Curves Showing Linear Relation Between Field Strength and Induction.

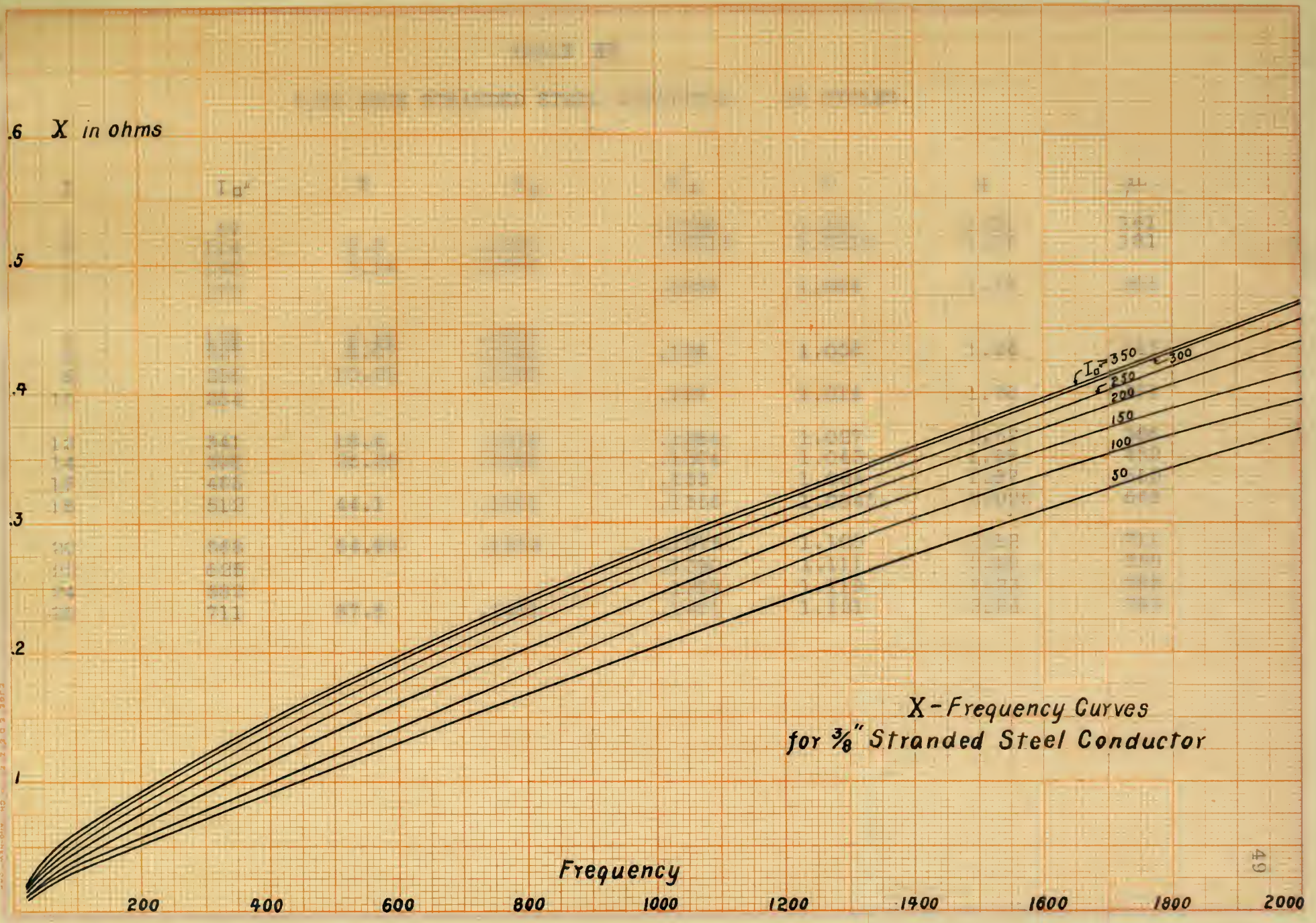
Curves Showing Linear Relation Between Field Strength and Induction.

Fig. 1. Curves Showing Linear Relation Between Field Strength and Induction.



Impedance - Current Density and
Reactance - Current Density Curves
for 3/8" Stranded Steel Conductor
Length 64' 0" Spacing 6"
Resistance .0493 ohms

Impedance - Current Diagram
Reactance - Current Diagram
Resistance - Current Diagram
Temp. at 0 min.
" " 5 "
" " 10 "
" " 15 "
" " 20 "
" " 25 "
" " 30 "
" " 35 "
" " 40 "
" " 45 "
" " 50 "
" " 55 "
" " 60 "
Impedance in ohms
Reactance in ohms
Resistance in ohms
Current in amperes
5000 cycles
4000 cycles
3000 cycles
2500 cycles
2000 cycles
1500 cycles
1000 cycles


X in ohms
.6
.5
.4
.3
.2
1
I_a
350
300
250
200
150
100
50
X-Frequency Curves
for 3/8" Stranded Steel Conductor
Frequency
200
400
600
800
1000
1200
1400
1600
1800
2000

for ⅜" Standard Steel Concrete
X - Frequency Curves
Frequency
Ohms in X
20
100
120
500
T=320
T=300

TABLE XX.

7/32 INCH STRANDED STEEL CONDUCTOR. 25 CYCLES.

I	$I_{a''}$	W	R_E	R'_E	K	q	μ
2	57			.1250	1.00+	1.01	161
4	114	2.0	.1250	.12515	1.0015	1.07	181
5	142	3.14	.1256				
6	170			.1255	1.004	1.15	205
7	199	6.16	.1257				
8	227	8.07	.1260	.126	1.008	1.24	243
9	256	10.25	.1266				
10	284			.127	1.016	1.38	301
12	341	18.4	.1278	.1284	1.027	1.52	366
14	398	25.25	.1288	.1304	1.043	1.69	452
16	455			.133	1.064	1.87	553
18	512	44.1	.1361	.1356	1.0845	2.025	649
20	568	54.95	.1373	.1375	1.100	2.12	711
22	625			.1390	1.111	2.19	758
24	682			.1399	1.119	2.23	786
25	711	87.6	.1401	.1401	1.121	2.24	793

TABLE XXI.

7/32 INCH STRANDED STEEL CONDUCTOR. 40 CYCLES.

I	$I_{0''}$	W	R_E	R'_E	K	q	μ
2	57			.12535	1.0025	1.11	122
4	114			.12595	1.0075	1.23	150
5	142	3.13	.1262				
6	170			.1270	1.016	1.375	187
7	199	6.26	.1277				
8	227			.12825	1.026	1.50	222
10	284			.1299	1.040	1.66	272
12	341			.1322	1.058	1.82	327
14	398			.1351	1.081	2.00	395
16	455	35.9	.1402	.1392	1.105	2.15	456
18	512	46.6	.1439	.1434	1.147	2.37	557
20	568	58.1	.1452	.1465	1.1715	2.485	610
22	625			.1488	1.190	2.56	647
23	654	79.5	.1502				
24	682			.15045	1.2035	2.615	675
25	711	94	.1504	.1510	1.208	2.63	684

TABLE XXII.

7/32 INCH STRANDED STEEL CONDUCTOR. 60 CYCLES.

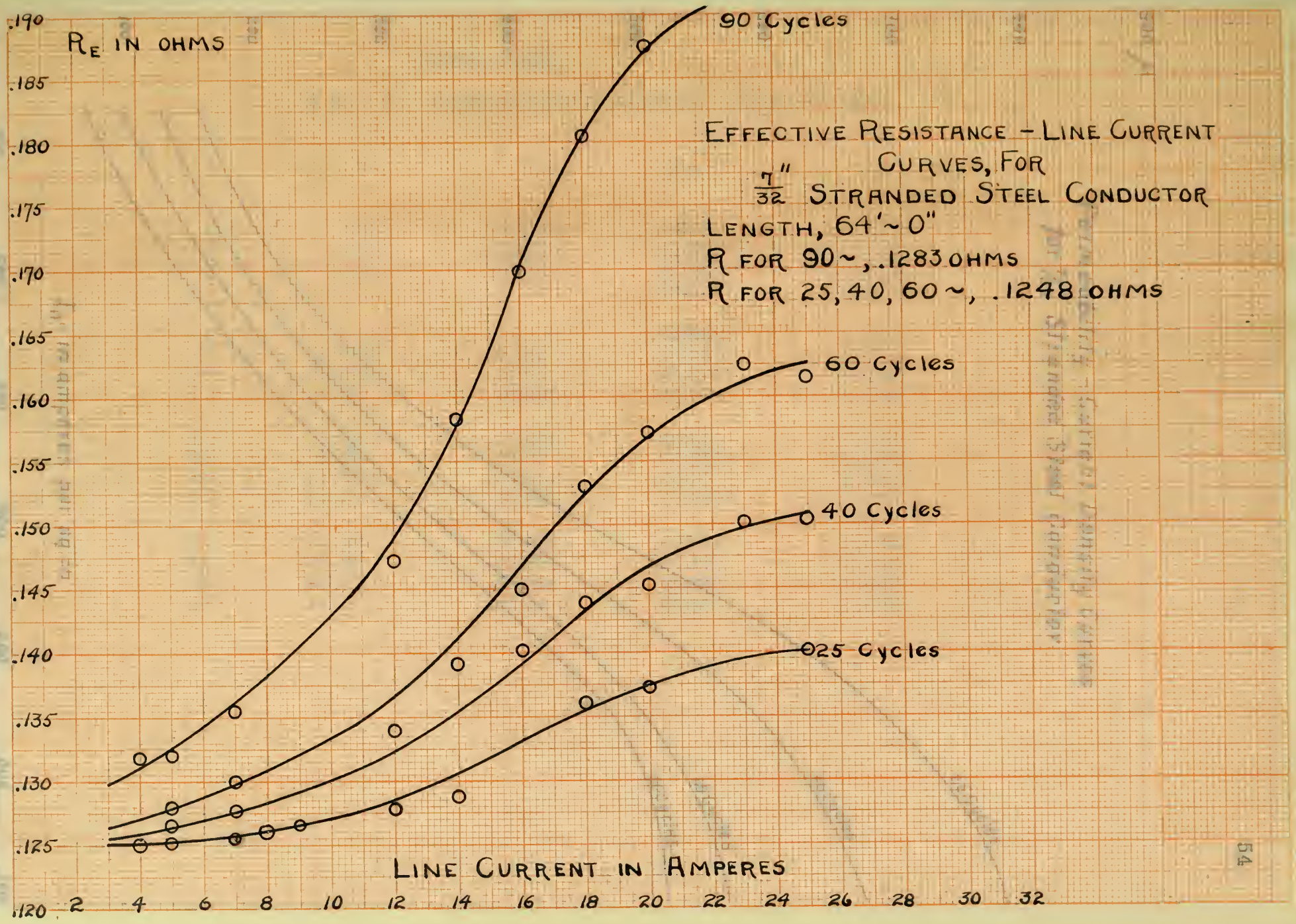
I	$I_{a''}$	W	R_E	R'_E	K	q	μ
2	57			.12585	1.007	1.22	98
4	114			.1271	1.017	1.39	127
5	142	3.2	.1280				
6	170			.12875	1.030	1.55	158
7	199	6.37	.130				
8	227			.13075	1.046	1.72	195
10	284			.1332	1.066	1.885	234
12	341	19.3	.134	.1365	1.092	2.07	282
14	398	27.3	.1392	.1408	1.126	2.27	339
16	455	37.1	.1449	.1467	1.174	2.495	410
18	512	49.6	.1530	.15285	1.223	2.69	477
20	568	62.9	.1572	.15695	1.256	2.81	520
22	625			.160	1.280	2.895	552
23	654	86	.1625				
24	682			.1621	1.297	2.945	571
25	711	100.9	.1615	.16265	1.301	2.958	584

TABLE XXIII.

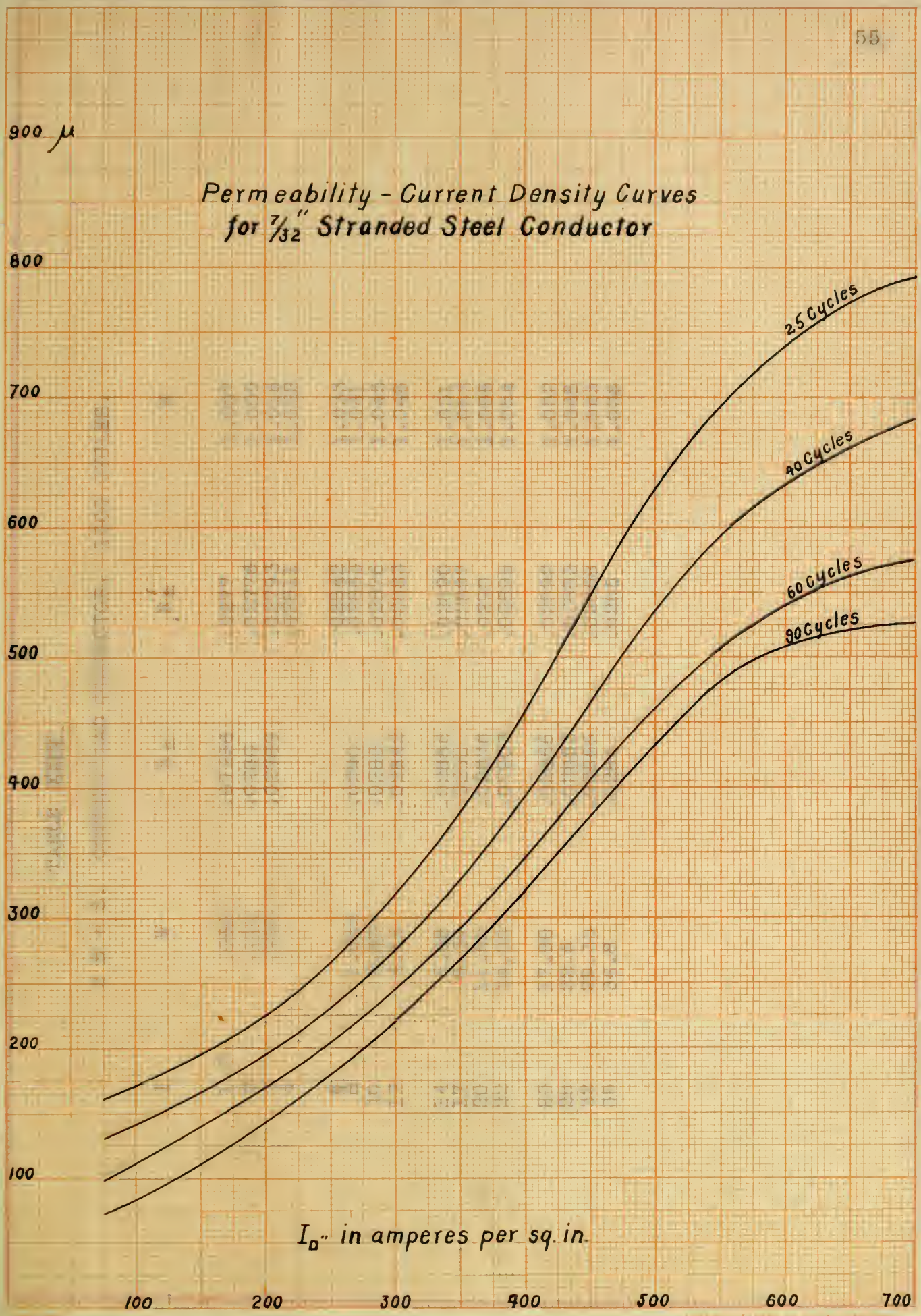
7/32 INCH STRANDED STEEL CONDUCTOR. 90 CYCLES.

I	$I_{\square''}$	W	R_E	R'_E	K	q	μ
2	57			.12915	1.007	1.22	64
4	114	2.11	.1318	.1312	1.023	1.465	92
5	142	3.30	.1320				
6	170			.1342	1.046	1.72	126
7	199	6.64	.1355				
8	227			.138	1.076	1.97	166
10	284			.14265	1.112	2.21	208
12	341	21.2	.1472	.1486	1.158	2.51	269
14	398	31.05	.1583	.158	1.231	2.723	3.7
16	455	43.5	.1698	.170	1.325	3.02	390
18	512	58.5	.1805	.1802	1.405	3.243	449
20	568	75	.1875	.1873	1.460	3.40	494
22	625			.1908	1.487	3.47	515
23	654	103	.1948				
24	682			.1925	1.500	3.505	525
25	711	122	.1952	.1927	1.509	3.533	534




R_E IN OHMS
.190
.185
.180
.175
.170
.165
.160
.155
.150
.145
.140
.135
.130
.125
.120
90 Cycles
60 Cycles
40 Cycles
25 Cycles
EFFECTIVE RESISTANCE – LINE CURRENT CURVES, FOR
$\frac{7}{32}$" STRANDED STEEL CONDUCTOR
LENGTH, 64'~0"
R FOR 90~, .1283 OHMS
R FOR 25, 40, 60~, .1248 OHMS
LINE CURRENT IN AMPERES
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32

Effective Resistance - Line Current
For 25 cycles
For 60 cycles
For 40 cycles
For 50 cycles
Curves Used in Plotting Results
Load, Watts
Power Factor
Line Current in Amperes
R.E. in ohms

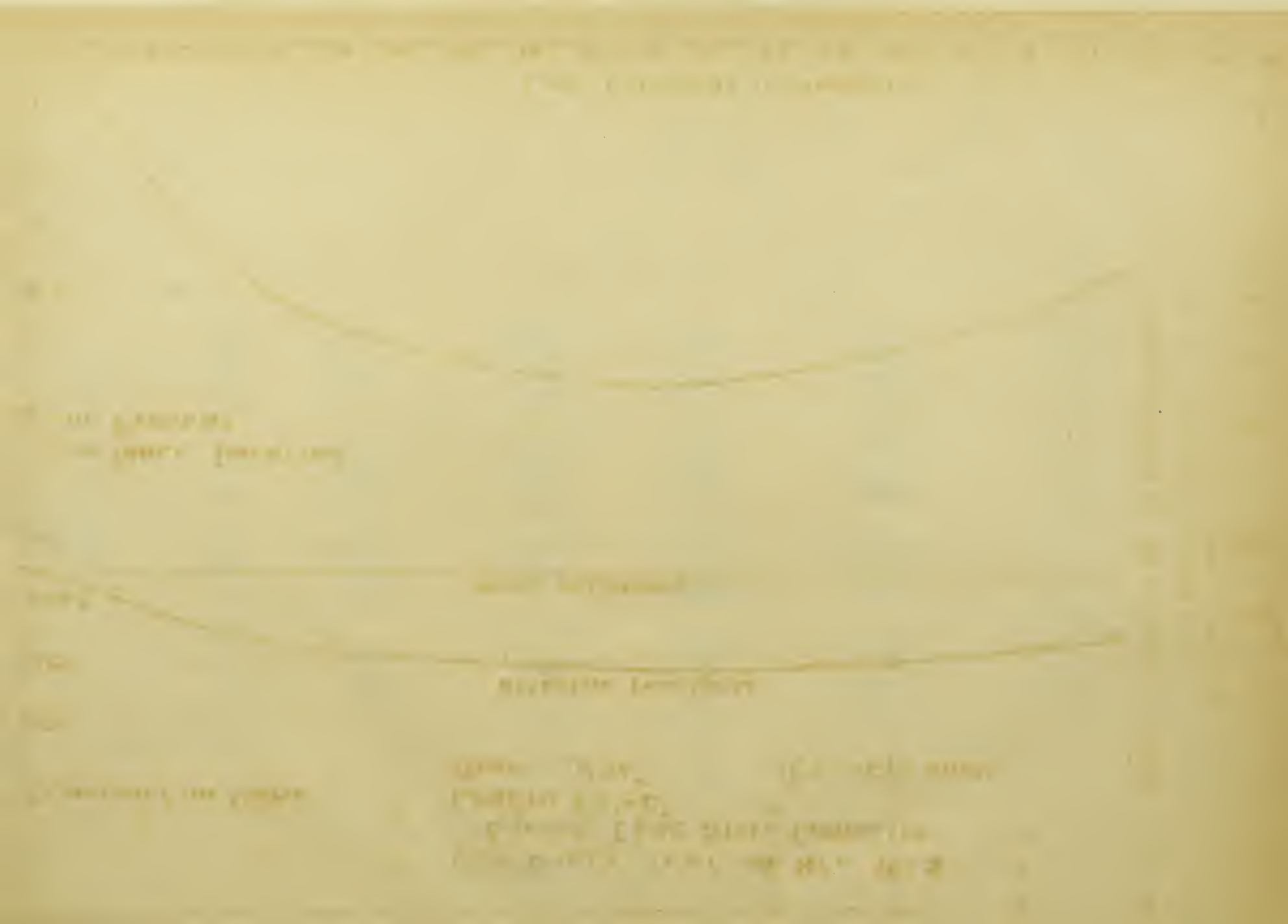

Permeability - Current Density Curves
for 7/32" Stranded Steel Conductor
900 μ
800
700
600
500
400
300
200
100
25 Cycles
40 Cycles
60 Cycles
90 Cycles
I_D" in amperes per sq. in.
100
200
300
400
500
600
700

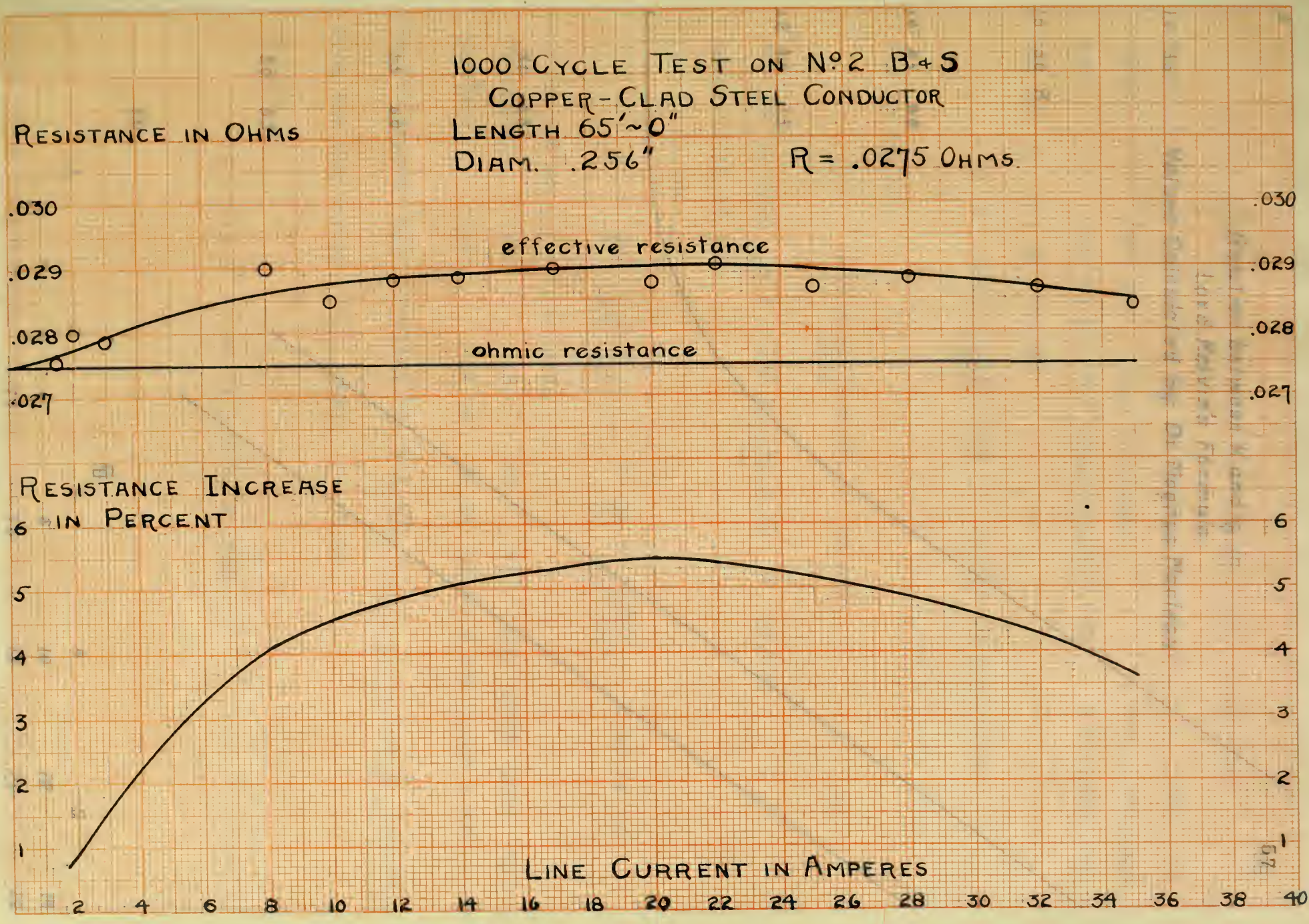
Permeability - Current Density Curves
for 7/16" Stranded Steel Conductor
μ
800
700
600
500
400
300
200
100
I_a - in amperes per sq in
100
200
300
400
500
600
700

TABLE XXIV.

2 B + S. COPPER-CLAD CONDUCTOR. 1000 CYCLES.

I	W	R_E	R_E'	K
1.5	.062	.07256	.0277	1.007
2	.112	.0280	.02776	1.009
3	.251	.02789	.02793	1.016
4			.02811	1.022
6			.02842	1.033
8	1.855	.0290	.02863	1.041
10	2.85	.0285	.02876	1.045
12	4.15	.02882	.02883	1.048
14	5.58	.02886	.02890	1.051
17	8.39	.0290	.02897	1.053
20	11.50	.02876	.0290	1.055
22	14.05	.02903	.02899	1.054
25	17.90	.02866	.02892	1.052
28	22.6	.02883	.02883	1.048
32	29.35	.02866	.02868	1.043
35	34.8	.0284	.0285	1.036




1000 Cycle Test on No. 2 B & S
Copper-Clad Steel Conductor
Length 65'~0"
Diam. .256"
R = .0275 Ohms.
Resistance in Ohms
.030
.029
.028
.027
effective resistance
ohmic resistance
Resistance Increase
in Percent
6
5
4
3
2
1
Line Current in Amperes
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40

1000 CYCLE TEST ON No. 2 B + S

ROTOR CONDUCTOR Steel Core Disc-Copper

Diam. 12.5" "D" 65 Height 7"

$R = .0150$ ohms

RESISTANCE IN OHMS

.050

.040

.030

.020

.010

Condition resistance switch

Condition simple

.0150

.0140

.0130

.0120

.0110

.0100

.0090

.0080

.0070

.0060

.0050

.0040

.0030

.0020

.0010

.0000

.0850

RESISTANCE INCREASE IN PERCENT

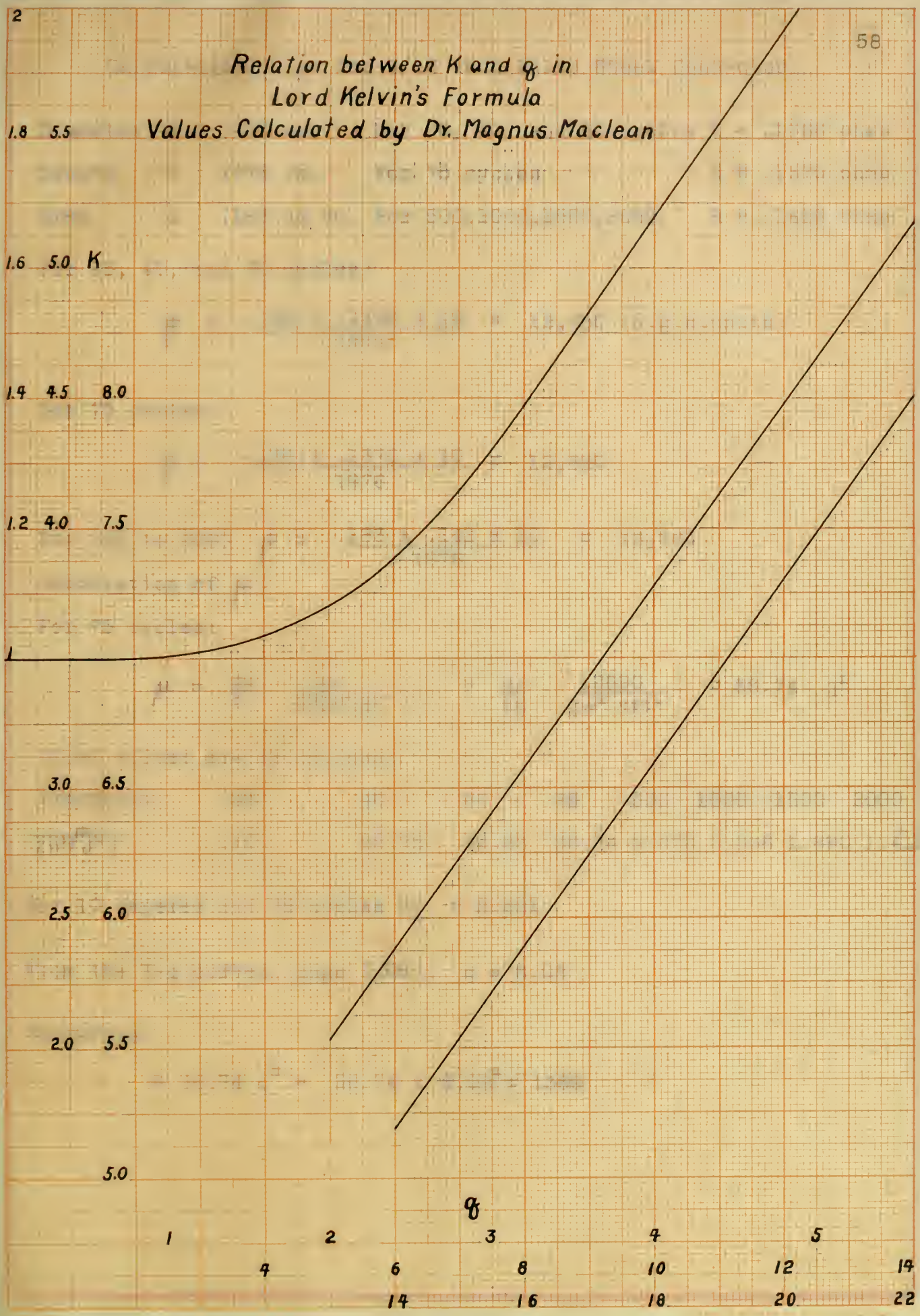
1 2 3 4 5 6

LINE CURRENT IN AMPERES

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

Relation between K and q_8 in Lord Kelvin's Formula

Values Calculated by Dr. Magnus Maclean



Relation between K and φ in
Lord Kelvin's Formula
Values Calculated by Dr Magnus Maclean

Calculation of ρ for 3/16 inch Solid Steel Conductor.

Diameter	d	.483 cm.	For 25, 40, and 60 cycles	R = .1366 ohms
Length	l	1976 cm.	For 75 cycles	R = .1370 ohms
Area	a	.183 sq cm.	For 500,1000,1500,2000	R = .1410 ohms

For 25, 40, and 60 cycles:

$$\rho = \frac{.183 \times .1366 \times 10}{1976} = 12{,}650 \text{ (c.g.s.units)}$$

For 75 cycles:

$$\rho = \frac{.183 \times .1370 \times 10}{1976} = 12{,}690$$

For 500 to 2000 $\rho = \frac{.183 \times .141 \times 10}{1976} = 13{,}060$

Calculation of μ:

For 75 cycles:

$$\mu = \frac{q^2}{f} \cdot \frac{\rho}{2\pi^2 d^2} = \frac{q^2}{75} \cdot \frac{12690}{2\pi^2 .483^2} = 36.74\ q^2$$

Other values are as follows:

Frequency	25	40	60	75	500	1000	1500	2000
$\frac{\rho}{2\pi^2 d^2 f}$	110	68.75	45.83	36.74	5.672	2.836	1.890	1.418

For 10 amperes and 75 cycles $R'_E = 2.421$

From the K-q curves, page (58), q = 6.08

Therefore

$$= 36.74\ q^2 = 36.74 \times 6.08^2 = 1359$$

Calculation of ρ for 3/8 inch Stranded Steel Conductor.

Length = 64'0" = 1951 cm. R = .0493 ohm.

Diameter per strand = .122"

Area per strand = $\pi r^2 = \pi .061^2 = .01169$ sq.in.

Area of 7 strands = .08183 sq.in. = .5278 sq.cm. = a

$$\rho = \frac{aR}{l} = \frac{.5278 \times .0493 \times 10}{1951} = 13{,}330 \text{ (c.g.s.units)}$$

"For all practical purposes, the diameter of a stranded conductor may be assumed the same as that of a solid conductor of the same area."- Standard Handbook for Electrical Engineers, Third Edition, page 43.

$a = \pi r^2$ $\qquad r = \sqrt{\frac{a}{\pi}} = \sqrt{\frac{.5278}{\pi}} = \sqrt{.1679} = .4098$ (cm.)

Equivalent diameter d =.8196 cm.

$$= \frac{q^2 \rho}{2\pi^2 d^2 f} = \frac{q^2}{f} \cdot \frac{13{,}330}{2 \times 9.87 \times .8196^2} = \frac{q^2}{f} \cdot 1{,}006$$

For 60 cycles:

$$\mu = q^2 \quad \frac{1006}{60} = q^2 \times 16.76$$

Other values are as follows:

Frequency	25	40	60	90	1000	2000
$\frac{\rho}{2\pi^2 d^2 f}$ =	40.25	25.15	16.76	11.23	1.006	.503

Calculation of ρ for 7/32 inch Stranded Steel Conductor.

Length = 64'4" = 1961 cm.

For 25, 40, and 60 cycles R = .1248 ohm

For 90 cycles R = .1283 ohm.

Diameter per strand = .080"

Area per strand = $\pi r^2 = \pi .04^2$ = .00527 sq. in.

Area of 7 strands = .03518 sq.in. = .2269 sq.cm. = a

For 25, 40, and 60 cycles:

$$\rho = \frac{a\,R}{l} = \frac{.2269 \times .1248}{1961} \times 10^9 = 14{,}430 \text{ (c.g.s. units)}$$

Use equivalent solid conductor.

$a = \pi r^3$ $\quad r = \sqrt{\frac{a}{\pi}} = \sqrt{\frac{.2269}{\pi}} = \sqrt{.0722} = .2687$ cm.

d = .5374 cm.

$$\mu = \frac{q^2 \rho}{2\pi^2 d^2 f} = \frac{q^2}{f} \cdot \frac{14430}{2\pi^2 \times .5374^2} = \frac{q^2}{f}\ 3{,}950$$

For 60 cycles:

$$\mu = q^2 \cdot \frac{3950}{60} = q^2 \times 65.8$$

For other frequencies:

Frequency	25	40	60	90
$\frac{2\pi^2 \rho d^2 f}{}$	158	98.7	65.8	43.7

Calculation of Z'_E and X.

Consider 3/8" stranded steel conductor, 18 amperes, 60 cycles.

From R_E - I curve, R'_E = .0765 ohm.

E observed = 1.526 volts.

$Z_E = \frac{E}{I} = \frac{1.526}{18} = .0848$ = effective impedance in ohms.

From Z_E - I curve, $Z'_E = .0840$

Tabulation of calculation is as follows:

I	18
R	.0765
Z	.0840
Z	705600
R	585225
X	120375
X	.0347

Calculation of Inductance due to Flux between Conductors.

The c.g.s unit of inductance for a one-turn coil is numerically equal to the flux produced by one c.g.s unit of current through the coil. A two-wire transmission line may be considered as a single turn conductor with two very long sides.

Consider only the total flux included between the conductors, disregarding the flux within the conductor itself.

Flux per cm. length of single wire and cgs unit of current

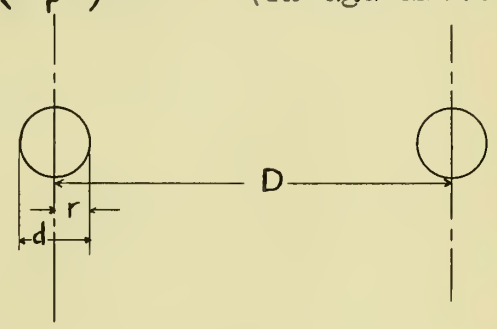
$= \Phi = 2 \log_\epsilon \left(\frac{D-r}{r}\right) = L$ = inductance in c.g.s units.

L for given length l in cm:

$L = 2\, l \log_\epsilon \left(\frac{D-r}{r}\right) = 2\, l \times 2.306 \log_{10} \left(\frac{D-r}{r}\right)$ (in c.g.s. units)

$L = 4.6\, l \log \left(\frac{D-r}{r}\right) \times 10^{-9}$

$D = 6" + 2r = 6" + d$



For 3/16" solid conductor:

$d = .190" = .4826$ cm.

$r = .095" = .2413$ cm.

$D = 6" + .19 = 6.19" = 15.72$ cm.

$l = 64'10" = 1976$ cm.

$\frac{D-r}{r} = \frac{15.7226 - .2413}{.2413} \doteq \frac{1548}{.2413} = 64.18$

$\log_{10}(64.18) = 1.8074$

$L = 4.6 \times 1976 \times 1.8074 \times 10^{-9} = .00001643$

$x = 2\pi f L$

For 60 cycles, $x = 2\pi\, 60 \times .00001643 = .006192$

Frequency	25	40	60	75	500	1000	1500	2000
X	.00258	.00413	.00619	.00774	.0516	.1032	.1548	.2064

Calculation for 3/8" Stranded Steel Conductor.

Equivalent d = .8196 cm.

r = .4098 cm.

D = 6" + d = 15.24 + .8196 = 16.06 cm.

l = 64'0" = 1951 cm.

$$\frac{D-r}{r} = \frac{16.06 - .410}{.4098} = \frac{15.65}{.4098} = 38.2$$

$\log_{10} 38.2 = 1.582$

$L = 4.6 \times 1951 \times 1.582 \times 10^{-9} = .0000142$ henry.

For 60~, $x = 2\pi 60 \times .0000142 = .005355$

Frequency	25	60	90	1000	2000
X	.00223	.005355	.00803	.0892	0.1785

Key to Notation.

R = ohmic resistance

R = effective resistance in ohms as calculated from $\frac{W}{I^3}$

R'= " " taken from curves of R

K = $\frac{R'}{R}$ = skin effect coefficient

W = watts consumed by the conductor

I = line current in amperes flowing in the conductor.

I = amperes per square inch, calculated from total area of conductor cross-section.

f = frequency in cycles per second.

q = variable in Lord Kelvin's formula.

μ = permeability

ρ = conductivity in absolute units

E = potential difference between ends of conductor in volts

Z_E= effective impedance in ohms, as calculated from $\frac{E}{I}$.

Z'_E= effective impedance, corrected value from Z_E curve

X = effective reactance in ohms, calculated.

V

CONCLUSIONS.

The curves indicate that in the conductors tested the permeability was a linear function of the current density over the range of current densities used in good practice (100 - 200 amperes per square inch).

The maximum permeability in the 3-16" solid steel conductor was 1640; in the 3-8" stranded steel cable the maximum was 400. In both conductors the maximum occurred at a current density of 400 amperes per square inch. In the 7-32" stranded cable the permeability had not reached a maximum even though the current density had risen to 700 amperes per square inch.

The initial value of permeability for the 3-16" solid conductor was 150, for the 3-8" stranded 25 - 50, for the 7-32" stranded 50 - 150. These values are comparable with Lord Rayleigh's value of 100 for soft iron under very small magnetic forces.

Every permeability curve has the same general shape, the chief difference being that the permeability increases with decrease of frequency.

The increase of the skin effect coefficient K with increase of frequency is greatest at frequencies below 100 cycles. Above 300 cycles K is approximately a linear function of frequency.

The decrease of internal inductance with increase of frequency causes the total inductance to increase less rapidly as higher frequencies are attained.

The reduction of skin effect by stranding is marked.

In spite of its greater area the 3-8" stranded cable shows less skin effect than the 3-16" solid wire at any given current density. A comparison at 60 cycles will illustrate. (See pages 13 and 33.)

	3-16" solid	3-8" stranded
I	K	K
100	1.37	1.09
200	1.99	1.40
400	2.25	1.75

The skin effect in the copper-clad steel wire is negligible compared to that in the steel conductors tested, being only 5 per cent at 1000 cycles, as compared with 200 - 400 per cent in the 3-8" cable.

APPENDIX I.

"When the current in a conductor is of varying intensity, the electromotive force arising from the induction of the current on itself is different in different parts of the section of the wire, being in general a function of the distance from the axis of the wire as well as of the time. If the cylindrical conductor be a solid mass in which electric currents are free to flow in obedience to electromotive force, the intensity of the current will not be the same at different distances from the axis of the cylinder, and the electromotive forces themselves will depend on the distribution of the current in the different cylindric strata of the wire.

"The vector potential H, the density of the current w, and the electromotive intensity at any point, must be considered as functions of the time and of the distance from the axis of the wire.

"The total current, C, through the section of the wire, and the total electromotive force E, acting round the circuit, are the variables, the relation between which are to be found in this discussion."

Consider the conductor as a straight circular cylinder, the axis of which is in the direction. The general expression for the component of current density and electromagnetic units, disregarding displacement current due to capacity effects, is

$$4\pi w = \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \qquad (1)$$

$4\pi w$ = component of current density,

where w = component of current,

L = X component of magnetic field intensity

M = y component of magnetic field intensity

This relation may be stated as follows: The electric current density is $\frac{1}{4\pi}$ times the curl of the magnetic field.

$$w = \frac{1}{4\pi}\left\{ \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right\}$$

Since the magnetic induction is the curl of its vector potential we may write:

$$a = \frac{\partial H}{\partial y} - \frac{\partial G}{\partial z} = \mu L \qquad (2)$$

$$b = \frac{\partial F}{\partial z} - \frac{\partial H}{\partial x} = \mu M \qquad (3)$$

where a and b are components of magnetic induction along the x and y axes

FGH are components of the vector potential of magnetic induction along the axes, sometimes called components of electromagnetic momentum, **U** at a point

μ = permeability = coefficient of magnetic induction such that $a = \mu L$, where L is a component of magnetic field intensity.

Since the vector potential H is at right angles to the z axis, $-\frac{\partial G}{\partial z}$ and $\frac{\partial F}{\partial z}$ are zero.

Then

$$a = \mu L = \frac{\partial H}{\partial y} \tag{4}$$

$$b = \mu M = -\frac{\partial H}{\partial x} \tag{5}$$

$$\frac{\partial M}{\partial X} = -\frac{1}{\mu} \frac{\partial^2 H}{\partial x^2} \tag{6}$$

$$\frac{\partial L}{\partial Y} = \frac{1}{\mu} \frac{\partial^2 H}{\partial y^2} \tag{7}$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = -\mu \frac{\partial M}{\partial x} + \mu \frac{\partial L}{\partial y} = -\mu\left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) = -\mu(4\pi w) = -4\pi\mu w$$

$$-4\pi\mu w = \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} \tag{8}$$

The right hand number of (8) is of the form of LaPlace's operator $\nabla^2 = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ acting on the vector H, but with the z term absent. Since a cylindrical conductor is being considered it is convenient to transform this expression to cylindrical coordinates, with the following result:

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 H}{\partial \phi^2} + \frac{\partial^2 H}{\partial z^2} \tag{9}$$

Since the vector potential is the same for all points at the same distance from the axis of the conductor, H is independent of the angle ϕ, and the third term may be omitted. Since H has no component along the z axis the fourth term may also be omitted.

Rewriting (8):

$$-4\pi\mu w = \frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial H}{\partial r} \qquad (10)$$

If a b c denote the components of magnetic induction, and l m n the direction-cosines of the normal to the given surface d S, the surface integral may be written

$$B = \iint (la + mb + nc)\, d\, S \qquad (11)$$

where B = magnetic induction through the surface d S.

But it is possible to determine the induction through this surface or closed curve dS by a process depending on the nature of that curve, and not on the normal to the surface. This may be done by finding a vector U, related to B, the magnetic induction, in such a way that the line integral of U, extended round the closed curve, is equal to the surface-integral of B, extended over any surface bounded by the closed curve.

Stokes' theorem states that the line integral round a closed curve of a vector quantity **U** with components X Y Z may be expressed in terms of the surface integral of the vector B provided the components a b c of B are related to X Y Z by the equations:

$$a = \frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z} \qquad (12)$$

$$b = \frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x} \qquad (13)$$

$$c = \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} \qquad (14)$$

The theorem may be written:

$$\int\left(X\frac{dx}{ds}+Y\frac{dy}{ds}+Z\frac{dz}{ds}\right)ds=\iint\left[\left(\frac{\partial z}{\partial y}-\frac{\partial Y}{\partial z}\right)l+\left(\frac{\partial X}{\partial z}-\frac{\partial Z}{\partial x}\right)m+\left(\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}\right)n\right]dS$$

The theorem holds always for the transformation from a line to a surface integral, but the transformation from a surface to a line integral is possible only when

$$\frac{da}{dx}+\frac{db}{dy}+\frac{dc}{d}=0. \qquad (15)$$

Equations (12) (13) (14) satisfy (15).

Thus

$$\iint(la+mb+nc)\,d\,S=\left(X\frac{dx}{ds}+Y\frac{dy}{ds}+Z\frac{dz}{ds}\right)ds \qquad (16)$$

If components F G H be substituted for X Y Z, (12) and (13) become identical with (2) and (3). Since F G H satisfy conditions (12) - (15) they may be substituted into (16), thus:

$$\iint(la+mb+nc)\,d\,S=\int\left(F\frac{dx}{ds}+G\frac{dy}{ds}+H\frac{dz}{ds}\right)ds \qquad (17)$$

But $B=\iint(la+mb+nc)\,d\,S$

Then $B=\iint(la+mb+nc)\,d\,S=\int\left(F\frac{dx}{ds}+G\frac{dy}{ds}+H\frac{dz}{ds}\right)d\,S$

The e.m.f. due to change of flux in a given time is:

$$-\frac{\partial B}{\partial t}=-\int\left(\frac{\partial F}{\partial t}\frac{dx}{ds}+\frac{\partial G}{\partial t}\frac{dy}{ds}+\frac{\partial H}{\partial t}\frac{dz}{ds}\right)ds$$

The term

$$-\int \frac{\partial H}{\partial t} \frac{dz}{ds}$$

may be written

$$-\int \frac{\partial H}{\partial t} dz.$$

$-\frac{\partial H}{\partial t}$ is the component of the electromotive force due to the change of the flux in the direction of the Z axis.

If in addition there exist a potential ψ corresponding to the part of the electromotive force which is independent of the current induction, we have

$\frac{\partial \psi}{\partial z} = Z = \frac{w}{l}$ = ohmic drop per unit length along the Z axis

Then $\frac{\partial \psi}{\partial z}$ is the electromotive force consumed by the resistance.

Thus the total electromotive force per differential length causing current along the Z axis is

$$-\left(\frac{\partial H}{\partial t} + \frac{\partial \psi}{\partial z}\right).$$

The vector potential H at each part of the circuit is a function of the time and the distance r of the point considered from the axis of the wire. Maxwell assumes that

$$H = S + T + T_1 r^2 + T_2 r^4 + T_3 r^6 + \ldots + T_n r^{2n} + \ldots \quad (25)$$

where S T T_1 etc., are functions of the time,

r = distance from the center of the cylindrical conductor.

$$\frac{dH}{dr} = 2\,T_1\,r + 4\,T_2 r^3 + 6 T_3 r^5 + \ldots + 2n T_n\, r^{2n-1} + \ldots \quad (26)$$

$$\frac{d^2H}{dr^2} = 2\,T_1 + 12\,T_2\,r^2 + 30 T_3\,r^4 + \ldots + 2n T_n\,(2n-1) r^{2n-2} + \ldots \quad (27)$$

From (10):

$$-4\pi\mu w = \frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\,\frac{\partial H}{\partial r}$$

$$-\pi\mu w = \frac{1}{4}\,\frac{\partial^2 H}{\partial r^2} + \frac{1}{4r}\,\frac{\partial H}{\partial r} \quad (28)$$

Substituting for $\frac{\partial^2 H}{\partial x^2}$ and $\frac{\partial H}{\partial r}$:

$$-\pi\mu w = \frac{1}{4}\,(2\,T_1 + 12\,T_2\,r^2 + 30\,T_3\,r^4 + \ldots + 2n\,T_n\,(2n-1)\,r^{2n-2} + \ldots$$

$$+ \frac{1}{4r}\,(2\,T_1\,r + 4\,T_2\,r^3 + 6\,T_3\,r^5 + \ldots +$$

$$+ 2n T_n\, r^{2n-1} + \ldots\ldots)$$

$$-\pi\mu w = \frac{T_1}{2} + 3\,T_2 r^2 + \frac{15}{2}\,T_3 r^4 + \ldots + \frac{N}{2}\,T_n\,(2n-1) r^{2n-2} + \ldots$$

$$\ldots$$

$$+ \frac{T_1}{2} + \frac{T_2}{r^2}\,T_2 r^2 + \frac{3}{2}\,T_3\,r^4 + \ldots + \frac{n}{2}\,T_n\,r^{2n-2} + \ldots$$

$$-\pi\mu w = T_1 + 4\,T_2 r^2 + 9\,T_3 r^4 + \ldots + \frac{n}{2}\,T_n (2n - 1 + 1) r^{2n-2} + \ldots$$

$$-\pi\mu w = T_1 + 2^2 T_2 r^2 + 3^2 T_3 r^4 + \ldots + n^2 T_n\, r^{2n-2} + \ldots \quad (29)$$

The component emf in the direction of z is $-\left(\frac{\partial H}{\partial t} + \frac{\partial \psi}{\partial z}\right)$. If ρ be the specific resistance of the material in the conductor and w the current density, the emf required is $\rho w = \frac{w}{\lambda}$, since $\rho = \frac{1}{\lambda}$ where λ is conductivity.

$$\frac{w}{\lambda} = -\left(\frac{\partial H}{\partial t} + \frac{\partial \psi}{\partial z}\right) \tag{30}$$

From equation (25):

$$\frac{\partial H}{\partial t} = \frac{\partial S}{\partial t} + \frac{\partial T}{\partial t} + r^2 \frac{\partial T_1}{\partial t} + r^4 \frac{\partial T_2}{\partial t} + \cdots + r^{2n} \frac{\partial T_n}{\partial t} + \cdots \tag{31}$$

$$-\frac{w}{\lambda} = +\frac{\partial \psi}{\partial z} + \frac{\partial S}{\partial t} + \frac{\partial T}{\partial t} + r^2 \frac{\partial T_1}{\partial t} + r^4 \frac{T_2}{\partial t} + \cdots + r^{2n-2} \frac{\partial T_{n-1}}{\partial t} + r^{2n} \frac{\partial T_n}{\partial t} + \cdots \tag{32}$$

Rewriting (29):

$$-w = \frac{T_1}{\pi\mu} + \frac{2^2 T_2 r^2}{\pi\mu} + \frac{3^2 T_3 r^4}{\pi\mu} + \cdots + \frac{n^2 T_n r^{2n-2}}{\pi\mu} + \frac{(n+1)^2 T_{n+1} r^{2n}}{\pi\mu} \tag{33}$$

Rewriting (32):

$$-w = \lambda \frac{\partial \psi}{\partial z} + \lambda \frac{\partial S}{\partial t} + \lambda \frac{\partial T}{\partial t} + \lambda r^2 \frac{\partial T_1}{\partial t} + \lambda r^4 \frac{\partial T_2}{\partial t} + \cdots + \lambda r^{2n-2} \frac{\partial T_{n-1}}{\partial t} + \lambda r^{2n} \frac{\partial T_n}{\partial t} + \cdots \tag{34}$$

Evaluating by equating the coefficients of like powers of r in (33) and (34):

$$\frac{T_1}{\pi\mu} = \lambda \frac{\partial \psi}{\partial z} + \lambda \frac{\partial S}{\partial t} + \lambda \frac{\partial T}{\partial t} \tag{35}$$

$$\frac{2^2 T_2}{\pi\mu} = \lambda \frac{\partial T_1}{\partial t} \tag{36}$$

$$\frac{3^2 T_3}{\pi\mu} = \lambda \frac{\partial T_2}{\partial t} \qquad (37)$$

$$\frac{n^2 T_n}{\pi\mu} = \frac{\lambda\, \partial T_{n-1}}{\partial t} \qquad (38)$$

Rewriting (35):

$$T_1 = \lambda\pi\mu\left(\frac{\partial\psi}{\partial z} + \frac{\partial S}{\partial t} + \frac{\partial T}{\partial t}\right) \qquad (39)$$

In equation (25) S was an arbitrary function of time. Let S be such a function of t that

$$\frac{\partial S}{\partial t} = \frac{-\partial\psi}{\partial z} \qquad (40)$$

where ψ is the potential corresponding to that part of the emf which is independent of current induction.

Substituting in (39) for $\frac{\partial S}{\partial t}$:

$$T_1 = \lambda\mu\pi\frac{\partial T}{\partial t} \qquad (41)$$

$$\frac{\partial T_1}{\partial t} = \lambda\pi\mu\frac{\partial^2 T}{\partial t^2} \qquad (42)$$

Rewriting (36), then substituting for $\frac{\partial T_1}{\partial t}$:

$$T_2 = \frac{\lambda\pi\mu}{2^2} \cdot \frac{\partial T_1}{\partial t} = \frac{\lambda\pi\mu}{2^2} \cdot \lambda\pi\mu\frac{\partial^2 T}{\partial t^2} = \frac{\lambda^2\pi^2\mu^2}{2^2}\frac{\partial^2 T}{\partial t^2} \qquad (43)$$

$$\frac{\partial T_2}{\partial t} = \frac{\lambda^2\pi^2\mu^2}{2^2} \cdot \frac{\partial^3 T}{\partial t^3} \qquad (44)$$

Rewriting (37), then substituting for $\frac{\partial T_2}{\partial t_2}$:

$$T_3 = \frac{\lambda\pi\mu}{3^2} \cdot \frac{\partial T_2}{\partial t} = \frac{\lambda\pi\mu}{3^2} \cdot \frac{\lambda^2\pi^2\mu^2}{2^2} \cdot \frac{\partial^3 T}{\partial t^3}$$

$$= \frac{\lambda^3\pi^3\mu^3}{2^2 \cdot 3^2} \cdot \frac{\partial^3 T}{\partial t^3} \qquad (45)$$

$$T_n = \frac{\lambda^n \pi^n \mu^n}{\lfloor n^2} \cdot \frac{\partial^n T}{\partial t^n} \qquad (46)$$

Substituting in (29) for T, T_1 T_2 ..... T_n :

$$-\pi\mu w = \lambda\pi\mu \frac{\partial T}{\partial t} + \frac{\lambda^2\pi^2\mu^2 r^2}{2^2} \frac{\partial^2 T}{\partial t^2} + \frac{3^2}{2^2 \cdot 3^2} \lambda^3\pi^3\mu^3 \frac{\partial^3 T}{\partial t^3} +$$

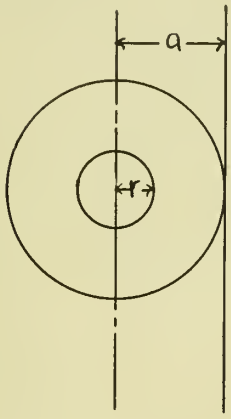
$$\ldots\ldots + \frac{n^2 \mu^n \pi^n \lambda^n}{\lfloor n^2} r^{2n-2} + \ldots\ldots\ldots$$

$$-\pi\mu w = \lambda\pi\mu \frac{\partial T}{\partial t} + \lambda^2\pi^2\mu^2 \cdot r^2 \frac{\partial^2 T}{\partial t^2} + \ldots\ldots + \frac{\mu^n \pi^n \lambda^n r^{2n-2}}{\lfloor (n-1)^2} + \ldots \qquad (47)$$

$$w = -\lambda \cdot \frac{\partial T}{\partial t} - \pi\lambda^2\mu \frac{\partial^2 T}{\partial t^2} r^2 + \ldots\ldots + \frac{\mu^{n-1} \pi^{n-1} \lambda^n}{\lfloor (n-1)^2} r^{2n-2} + \ldots\ldots \qquad (48)$$

If γ be the total current in the conductor,

$$\gamma = \int_{r=0}^{r=a} w\, 2\pi r\, dr \qquad (49)$$



Substituting for w in (49):

$$\gamma = 2\pi \int_0^a -\lambda \frac{\partial T}{\partial t} r\, dr - 2\pi \int_0^a \mu\pi\lambda^2 \frac{\partial^2 T}{\partial t^2} r^2 r\, dr + \ldots\ldots$$

$$\ldots\ldots - 2\pi \int \frac{\mu^{n-1} \pi^{n-1}}{\lfloor (n-1)^2} \lambda^n r^{2n-2} + \ldots\ldots$$

$$\gamma = -\pi\lambda \frac{\partial T}{\partial t} \frac{a^2}{2} - 2\pi^2\mu\lambda^2 \cdot \frac{\partial^2 T}{\partial t^2} \frac{a^4}{4} + \ldots\ldots \qquad (50)$$

Let K = conductivity of conductor for unit length.

$$K = \frac{\pi a^2}{\rho} = \pi a^2 \lambda \qquad (51)$$

$$\gamma = -K \frac{\partial T}{\partial t} - 2\frac{K^2}{2^2} \mu \frac{\partial^2 T}{\partial t^2} + \ldots\ldots + n \frac{\mu^{n-1} K^n}{\lfloor n^2} \frac{\partial^n T_n}{\partial t_n} + \ldots\ldots \qquad (52)$$

$$-\mu\gamma = K\mu\frac{\partial T}{\partial t} + \frac{2K^2\mu^2}{1^2\times 2^2}\frac{\partial^2 T}{\partial t^2} + \cdots\cdots + \frac{n\mu^n K^n}{\lfloor n^2}\frac{\partial^n T}{\partial t^n} + \ldots \quad (53)$$

The vector potential due to a small magnet is proportional to the strength of magnet m. It is evident that a conductor carrying current may be substituted for the magnet. Then the vector potential will be proportional to the total current in the wire. Outside the conductor the vector potential depends not on the distribution of current within the conductor but on the total current in the conductor. Hence, at the surface of the conductor let $H = A\gamma$, where A is a proportionality factor to be determined.

Rewriting (25), remembering that a is the value of r at the surface,

$$A = S + T + T_1 a^2 + T_2 a^4 + \cdots + T_n a^{2n} + \ldots \quad (54)$$

Substituting the values of $T_1\ T_2\ T_3\ \ldots\ldots\ T_n$ in (41) (43) (45) (46)

$$A = S + T + \mu\pi\lambda\frac{\partial Ta^2}{\partial t} + \frac{\mu^2\pi^2\lambda^2}{1^2\times 2^2}\frac{\partial^2 T}{\partial t^2}a^4 \quad (55)$$

$$+\frac{\mu^3\pi^3\lambda^3}{1^2\cdot 2^2\cdot 3^2}\frac{\partial^3 T}{\partial t^3}a^6 + \cdots + \frac{\mu^n\pi^n\lambda^n}{\lfloor n^2}\frac{\partial^n T}{\partial t^n}a^{2n} + \ldots$$

$$A\gamma = S + T + K\mu\frac{\partial T}{\partial t} + \frac{K^2\mu^2}{1^2\cdot 2^2}\frac{\partial^2 T}{\partial t^2} + \frac{K^3\mu^3}{1^2\cdot 2^2\cdot 3^2}\frac{\partial^3 T}{\partial t^3} + \cdots\cdots + \quad (56)$$

$$+\frac{K^n\mu^n}{\lfloor n^2}\frac{\partial^n T}{\partial t^n} + \cdots\cdots$$

Using the symbolic operator $\frac{d}{dt}$:

$$A\gamma = S + \left[1 + K + \frac{d}{dt} + \frac{K^2\mu^2}{1^2\cdot 2^2}\frac{d^2}{dt^2} + \frac{K^3\mu^3}{1^2\cdot 2^2\cdot 3^3}dt^3 + \cdots\cdots\right.$$

$$\left.\cdots + \frac{K^n\mu^n}{\lfloor n^2}\frac{d^n}{dt^n} + \cdots\cdots\right]T \quad (57)$$

Let $\phi(x) = \sum_{n=0}^{n=\infty} \frac{x^n}{\lfloor n^2} = 1 + \frac{x}{1^2} + \frac{x^2}{1^2 \cdot 2^2} + \frac{x^3}{1^2\ 2^2\ 3^2} + \cdots$

$$+ \cdots\cdots + \frac{x^n}{\lfloor n^2} + \cdots \qquad (58)$$

It is well to note that $\lfloor 0 = 1$.

$$\text{If } x = \left(K\mu \frac{d}{dt}\right) T \qquad (59)$$

$$A\gamma - S = \phi(x) = \phi\left(\mu K \frac{d}{dt}\right) T \qquad (60)$$

$$S = A\gamma - \phi\left(\mu K \frac{d}{dt}\right) T \qquad (61)$$

$$\frac{dS}{dt} = A \frac{d\gamma}{dt} - \phi\left(\mu K \frac{d}{dt}\right) \frac{dT}{dt} \qquad (62)$$

Then $\phi'(x) = \frac{d}{dx} \phi(x) = 1 + \frac{2x}{1^2 \cdot 2^2} + \frac{3x^2}{1^2 \cdot 2^2 \cdot 3^2} + \cdots\cdots$

$$= \sum_{n=0}^{n=\infty} \frac{n x^{n-1}}{\lfloor n^2} \qquad (63)$$

From (53):

$$\gamma = \left[- K \frac{d}{dt} - 2 \frac{k^2 \mu}{1^2 \cdot 2^2} \frac{d^2}{dt^2} + \cdots - \frac{n \mu^{n-1} K^n}{\lfloor n^2} \frac{d^n}{dt^n} \right] T \qquad (64)$$

$$\gamma = - K \frac{d}{dt} \left[1 + \frac{2k\mu}{1^2 \cdot 2^2} \frac{d}{dt} + \cdots\cdots + \frac{n \mu^{n-1} K^{n-1}}{\lfloor n^2} \frac{d^{n-1}}{dt^{n-1}} \right] T \qquad (65)$$

$$\gamma = - K \phi'\left(\mu K \frac{d}{dt}\right) \frac{dT}{dt} \qquad (66)$$

From (62): (67)

$$\frac{dT}{dt} = \frac{A \frac{d\gamma}{dt} - \frac{dS}{dt}}{\phi'\left(\mu K \frac{dt}{dt}\right)}$$

From (66):

$$\frac{dI}{dt} = \frac{\gamma}{-K\phi'\left(\mu K \frac{d}{dt}\right)} \qquad (68)$$

Then

$$\frac{A\frac{d\gamma}{dt} - \frac{dS}{dt}}{\phi\left(\mu K \frac{d}{dt}\right)} = \frac{\gamma}{-K\phi'\left(\mu K \frac{d}{dt}\right)} \qquad (69)$$

$$A\frac{d\gamma}{dt} - \frac{dS}{dt} = \frac{\gamma\,\phi\left(\mu K \frac{d}{dt}\right)}{-K\phi'\left(\mu K \frac{d}{dt}\right)} \qquad (70)$$

$$\frac{dS}{dt} = A\frac{d\gamma}{dt} + \frac{\gamma\phi\left(\mu K \frac{d}{dt}\right)}{K\phi'\left(\mu K \frac{d}{dt}\right)} \qquad (71)$$

But it has been assumed that

$$\frac{\partial \Psi}{\partial z} = -\frac{dS}{dt}$$

Consider that the conductor is of uniform resistance since the potential $\frac{\partial \Psi}{\partial z}$ is independent of any inductive action of the current and therefore applies to uniform current distribution.

The fall of potential in the wire

$$= w.\ \text{area}\ \frac{\rho\, dz}{\text{area}} = w\rho dz = \frac{w\, dz}{\lambda} \qquad (72)$$

For unit distance $dz = 1$

Potential difference per unit distance $= \frac{w}{x} = -\frac{\partial \psi}{\partial t} = \frac{ds}{dt}$ (73)

But $w = \frac{\text{total current}}{\text{cross section}} = \frac{\gamma}{\pi a^2}$ (74)

$$\frac{w}{x} = \frac{\gamma}{\pi a^2 \lambda} = \frac{\gamma}{K} = \frac{ds}{dt} \tag{75}$$

If total emf = E, and length in cm = 1, the drop along the axis per cm. distance = $\frac{E}{l}$ (76)

E = the emf. due to other causes than the induction of the current upon itself.

Substituting in (71):

$$\frac{E}{l} = A \frac{d\gamma}{dt} + \frac{1}{K} \frac{\phi\left(\mu K \frac{d}{dt}\right)}{\phi'\left(\mu K \frac{d}{dt}\right)} \gamma \tag{77}$$

$$E = A\, l \frac{d\gamma}{dt} + \frac{1}{K} \frac{\phi\left(\mu K \frac{d}{dt}\right)}{\phi'\left(\mu K \frac{d}{dt}\right)} \gamma \tag{78}$$

If a current be periodic according to the simple harmonic law it may be written

$$i = I \epsilon^{jpt} \tag{79}$$

where $p = 2\pi f$

$$\frac{di}{dt} = j p I \epsilon^{jpt} = jpi \tag{80}$$

$$\left(\frac{d}{dt}\right) i = (j p) i$$

$$\left(\frac{d}{dt}\right) = j p \tag{81}$$

Substituting for $\frac{d}{dt}$ in (78):

$$E = A\, l\, j\, p\, \gamma + \frac{1}{K} \frac{\phi(\mu K j p)}{\phi'(\mu K j p)} \gamma \tag{82}$$

$$\frac{\phi(x)}{\phi'(x)} = 1 + x + \frac{x^2}{4} + \frac{x^3}{36} + \ldots\ldots \div 1 + \frac{x}{2} + \frac{x^2}{12} + \ldots\ldots$$

$$= 1 + \frac{x}{2} - \frac{x^2}{12} + \frac{x^3}{48} - \frac{x^4}{180} + \frac{13\,x^5}{8640} + \ldots\ldots\ldots \quad (83)$$

Substituting ($\mu K\ p\ j$) for x, separating the real from the imaginary part, and remembering that $j^2 = -1$

$$\frac{\phi(\mu K\ p\ j)}{\phi'(\mu K\ p\ j)} = 1 + \frac{\mu^2 K^2 p^2}{12} - \frac{\mu^4 K^4 p^4}{180} + \ldots$$

$$+ j\left(\frac{\mu K p}{2} - \frac{\mu^3 K^3 p^3}{48} + \frac{13}{8640}\mu^5 K^5 p^5 + \ldots\ldots\right) \quad (84)$$

If m denotes the real part, and n the imaginary,

$$E = j\,a\,\ell\,p\,\gamma + \frac{\ell\,\gamma}{K}\,(m + j\,n) \quad (85)$$

But $\frac{\ell}{K} = \frac{\ell}{\pi a^2 \lambda} = \frac{\ell \rho}{\pi a^2} = R =$ actual ohmic resistance (86)
(for steady currents) of conductor length ℓ.

$$E = R\,\gamma\,m + j\,a\,l\,p\,\gamma + \frac{j\,l\,n\,\gamma}{K} = R\,m\,\gamma + j\,l\,\gamma\left(ap + \frac{n}{K}\right) \quad (87)$$

$$= R\left(1 + \frac{\mu^2 K^2 p^2}{12} - \frac{\mu^4 K^4 p^4}{180} + \ldots\ldots\right)\gamma +$$

$$+ j\,\ell\,\gamma\left[ap + \frac{1}{K}\left(\frac{K\mu p}{2} - \frac{\mu^3 K^3 p^3}{48} + \frac{13}{8640}\mu^5 K^5 p^5 + \ldots\right)\right] \quad (88)$$

But $\frac{\ell}{K} = R.\quad K = \frac{\ell}{R}$

$$E = R\left(1 + \frac{\mu^2 \ell^2 p^2}{12\,R^2} - \frac{\mu^4 \ell^4 p^4}{180\,R^4} + \ldots\ldots\right)\gamma$$

$$+ j\,\ell\,\gamma\,p\left[a + \mu\left(1 - \frac{\mu^2 \ell^2 p^2}{48\,R^2} + \frac{13}{8640}\,\frac{\mu^4 \ell^4 p^4}{R^4} + \ldots\right)\right] \quad (89)$$

This equation is of the form

$$E = R'\gamma + j\,p\,L'\gamma = R'\gamma + L'\,\frac{d\gamma}{dt} \qquad (90)$$

$$\left[jp = \frac{d}{dt} \qquad J\,p\,\gamma = \frac{d\gamma}{dt}\right]$$

where

$$R' = R\left(1 + \frac{\mu^2 l^2 p^2}{12\,R^2} - \frac{\mu^4 l^4 p^4}{180\,R^4} + \ldots\ldots\right) \qquad (91)$$

$$L' = l\left[a + \mu\left(1 - \frac{\mu^2 l^2 p^2}{48\,R^2} + \frac{13}{8640}\,\frac{\mu^4 l^4 p^4}{R^4}\right)\right] \qquad (92)$$

R' represents the effective resistance and L' the effective self induction.

If the rate of alternation be very low, so that p is small, the resistance R' becomes approximately R, and the self induction becomes $L' = l\,(A + \frac{1}{2}\mu)$, the values for steady currents.

K = skin effect coefficient = $1 + \frac{\mu^2 l^2 p^2}{12\,R^2} - \frac{\mu^4 l^4 p^4}{180\,R^4} + \ldots$ (93)

A = inductance between wires, a logarithmic expression independent of skin effect.

$\frac{1}{2}\mu$ = inductance for uniformly distributed current.

"As the frequency of alternation increases from zero to infinity the resistance rises steadily, and the self induction decreases. At slow rates of alternation the distribution of current, being such as to make the resistance a minimum, is uniform over the section; and this distribution,

since it involves magnetization of the outer parts of the cylinder, leads to considerable self induction, especially in iron. When the rate of alternation is very rapid the endeavor is to make the self induction a minimum irrespective of resistance. This object is attained by concentration of the current in the outer layers. The magnetization of the conductor is thus more and more avoided, but at the expense of increased resistance.

"The ultimate value of R' and L' when p is very great may be arrived at analytically. The series $\phi(x)$ (58) may then be replaced by $\frac{1}{2\sqrt{\pi}} \frac{e^{2\sqrt{x}}}{x^{\frac{1}{4}}}$ from which

$$\frac{\phi(x)}{\phi'(x)} = x^{\frac{1}{2}}$$

or $$\frac{\phi(i\,p\,K\,\mu)}{\phi'(i\,p\,K\,\mu)} = \sqrt{(\tfrac{1}{2}\,p\,K\,\mu)} \cdot (1+i)$$

Accordingly the limiting values of resistance and self induction are given by

$$R' = R\sqrt{(1/2\;p\;k\;\mu)} = \sqrt{(\tfrac{1}{2}p\,l\,\mu\,R} \qquad (94)$$

$$L' = l\left\{A + \frac{\sqrt{\mu}}{\sqrt{2pK}}\right\} = l\left\{A + \sqrt{\frac{\mu R}{2\,p\,l}}\right. \qquad (95)$$

R' increases without limit with p, while L' tends to the finite value $l\,A$."

-Lord Rayleigh , Phil. Mag. May '86, pg. 389-390.

In the notation of Bessel's functions, if x be any number

$$J_0(jx) = 1 + \frac{x^2}{2^3} + \frac{x^4}{2^2 4^2} + \frac{x^6}{2^2 4^2 6^2} + \cdots = \phi\left(\frac{x^2}{2^2}\right) \qquad (96)$$

Let $\left(\frac{x^2}{2^2}\right) = jk\mu p$ (97)

$x = 2\sqrt{j\,k\,\mu p} = q\sqrt{j}$ (98)

where $q = 2\sqrt{k\,\mu p}$ (99)

Then $J_o(jx) = J_o\left(j\,q\sqrt{j}\right) = \phi\,(j\,k\,\mu p) =$

$$= 1 + \frac{jq^2}{2^2} - \frac{q^4}{2^2 \cdot 4^2} - \frac{iq^6}{2^2 \cdot 4^2 \cdot 6^2} + \frac{q^8}{2^2 4^2 6^2 8^2} + \cdots\cdots \quad (100)$$

Lord Kelvin calls the real part of this expression berq, and the imaginary part beiq. [be-Bessel, r=real]

Then $\phi(j\,k\,\mu p) = \text{ber}\,q + j\,\text{bei}\,q$ (101)

$$\frac{\phi(j\,k\,\mu p)}{\phi'(j\,k\,\mu p)} = \frac{\text{ber}\,q + j\,\text{bei}\,q}{(\text{ber}'q + j\,\text{bei}'q)\,\frac{dq}{d(j\,k\,\mu p)}} \quad (102)$$

where $\text{ber}'q = \frac{d\,\text{ber}\,q}{dq}$ and $\text{bei}'q = \frac{d\,\text{bei}\,q}{dq}$

Note: if $\phi(x) = f(y)$

$$\frac{d\,\phi(x)}{dx} = \frac{df(y)}{dx} = \frac{df(y)}{dy}\,\frac{dy}{dx}$$

$$\phi'(x) = f'(y)\,\frac{dy}{dx}$$

Suppose $F(j\,k\,\mu p) = \text{ber}\,q$ and $\psi(j\,k\,\mu p) = j\,\text{bei}\,q$

$$F(j\,k\,\mu p) = \text{ber}'q\,\frac{dq}{d(j\,k\,\mu p)} \qquad \psi'(j\,k\,\mu p) = j\,\text{bei}'q\,\frac{dq}{d(j\,k\,\mu p)}$$

But $\phi(jk\mu p) = F(jk\mu p) + \psi(jk\mu p)$

$$\phi'(jk\mu p) = (\text{ber}'q + j\,\text{bei}'q)\frac{dq}{d(jk\mu p)} \qquad (103)$$

$$\frac{dq}{d(jk\mu p)} = \frac{d\left(2\sqrt{k\mu p}\right)}{d(jk\mu p)} \qquad (104)$$

Let $jk\mu p = z$. $\quad k\mu p = \frac{z}{j}$ (105)

$$q = 2\sqrt{k\mu p} = 2\sqrt{\frac{z}{j}} \qquad (106)$$

$$\frac{dq}{d(jk\mu p)} = \frac{d\,2\sqrt{\frac{z}{j}}}{dz} = \frac{2\,d\left(z^{\frac{1}{2}}\right)}{\sqrt{j}\,dz} =$$

$$\frac{2}{\sqrt{j}}\,\frac{\frac{1}{2}z^{-\frac{1}{2}}\,dz}{dz} = \frac{1}{\sqrt{jz}} = \frac{1}{\sqrt{j^2 k\mu p}} = \frac{1}{j\sqrt{k\mu p}}$$

$$= \frac{1}{j\frac{q}{2}} = \frac{2}{jq} \qquad (107)$$

Then $\frac{\phi}{\phi'} = \frac{jq}{2} \cdot \frac{\text{ber}\,q + j\,\text{bei}\,q}{\text{ber}'q + j\,\text{bei}'q}$ (108)

Multiplying by $\quad \frac{\text{ber}'q - j\,\text{bei}'q}{\text{ber}\,q - j\,\text{bei}\,q}$

$$\frac{\phi}{\phi'} = \frac{jq}{2}\,\frac{(\text{ber}\,q + j\,\text{bei}\,q)(\text{ber}'q - j\,\text{bei}'q)}{(\text{ber}'q)^2 + (\text{bei}'q)^2}$$

$$= \frac{jq}{2}\,\frac{\text{ber}\,q\,\text{ber}'q + j\,\text{bei}\,q\,\text{ber}'q - j\,\text{ber}\,q\,\text{bei}'q + \text{bei}\,q\,\text{bei}'q}{(\text{ber}'q)^2 + (\text{bei}'q)^2}$$

$$\frac{\phi}{\phi'} = \frac{q}{2}\left[\frac{\text{ber } q\ \text{bei}'q - \text{bei } q\ \text{ber}'q}{(\text{ber}'q)^2+(\text{bei}'q)^2} + j\,\frac{\text{ber } q\ \text{ber}'q + \text{bei } q\ \text{bei}'q}{(\text{ber}'q)^2+(\text{bei}'q)^2}\right.$$

Let m denote the real part, n the imaginary part.

From (82):

$$E = A\,l\,j\,p\gamma + \frac{l}{k}\,\frac{\phi}{\phi'}\,\frac{(\mu k\ j\ p)}{(\mu k\ j\ p)}\gamma = A\,l\,j\,p\gamma + \frac{l}{k}\,\frac{q}{2}\,(m+j\,n)\gamma$$

$$E = \frac{l}{k}\,\frac{q}{2}\,m\gamma + j\left(A\,l\,p\gamma + \frac{l}{k}\,\frac{q}{2}\,n\,\gamma\right)$$

But $R = \frac{l}{k}$.

$$E = R\,\frac{q}{2}\,m\gamma + j\gamma\left(A\,l\,p + \frac{1}{k}\,\frac{q}{2}\,n\right)$$

$$E = R\,\frac{q}{2}\,m\gamma + j\,p\gamma\left(A\,l + \frac{l\,q\,n}{2\,k\,p}\right)$$

This is of the form

$$E = R'\gamma + j\,p\,L'\gamma = R'\gamma + L'\,\frac{d\gamma}{dt} \quad \left[\text{since } jp\,(\gamma) = \frac{d}{dt}\,(\gamma)\right]$$

$$R' = R\,\frac{q}{2}\,m = R\,\frac{q}{2}\,\frac{\text{ber } q\ \text{bei}'q - \text{ber}'q\ \text{bei } q}{(\text{ber}'q)^2 + (\text{bei}'q)^2}$$

$$L' = \frac{l\,q\,n}{2\,k\,p} + A\,l = \frac{l\,q}{2Kp}\left(\frac{(\text{ber}'q\ \text{ber } q + \text{bei } q\ \text{bei}'q}{(\text{ber}'q)^2 + (\text{bei}'q)^2}\right) + Al$$

In this equation

$$Q = 2\sqrt{K\mu p} = 2\sqrt{K\mu 2\pi f}$$

$$\text{and } K = \frac{l}{R} = \frac{l}{\frac{\rho l}{\frac{\pi d^2}{4}}} = \frac{\pi d^2}{4\rho}$$

$$q = 2\sqrt{\frac{\pi d^2}{4\rho}\mu\, 2\pi f} = \frac{\cancel{2}\pi d}{\cancel{2}}\sqrt{\frac{2\mu f}{\rho}}$$

$$2\mu f = \frac{\rho q^2}{\pi^2 d^2}$$

$$= \frac{q^2\rho}{2\pi^2 d^2 f}$$

Considering K as a function of q Dr. Magnus McLean has calculated the following table of numerical values for K corresponding to values of q ranging from 0 to 20.

q	$K = \frac{R}{R}$	q	$K = \frac{R}{R}$
0.0	1.0000	4.5	1.8628
0.5	1.0000	5.0	2.0430
1.0	1.0001	5.5	2.2190
1.5	1.0258	6.0	2.3937
2.0	1.0805	8.0	3.0956
2.5	1.1747	10.0	3.7940
3.0	1.3180	15.0	5.5732
3.5	1.4920	20.0	7.2350

Plotting these values shows that the critical values of K are from q = 0 to q = 5. For q 5 the curve is a straight line, showing that K varies directly as q.

From experimental data the value of K is determined. From the table the corresponding value of q is found. Then all the terms of the right hand member are known, and the value of permeability can be obtained.

BIBLIOGRAPHY.

"The Principles of Electric Wave Telegraphy and Telephony" J.A.Fleming.

"The Alternating Current Transformer" Vol. 1, Third Edition J.A.Fleming.

"Transient Electric Phenomena and Oscillations" Steinmetz.

"Absolute Measurements in Electricity and Magnetism" Vol.II A. Gray.

"Electricity and Magnetism" Vol. II, Third Edition, Maxwell.

"Alternating Currents" Russell.

"Elements of Electricity and Magnetism" J. J. Thomson.

"Recent Researches in Electricity and Magnetism" J. J. Thomson.

"Electromagnetic Theory" Heaviside.

"Standard Handbook for Electrical Engineers", Third Edition, McGraw-Hill Book Company.

"The Self-induction and Resistance of Straight Conductors", Lord Rayleigh, Philosophical Magazine, May 1886, Vol. XXI, pg. 381.

"The Effective Resistance and Inductance of a Concentric Main and Methods of Computing the Ber. and Bei. and allied Functions". Dr. A. Russell, Philosophical Magazine, April 1909, pg. 524.

UNIVERSITY OF ILLINOIS-URBANA
3 0112 079827215